Conformal-07

Q.N1. Find the domain of the function

$$y = \sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$$

Soln:

$$\frac{5x-x^2}{4} > 0 \Rightarrow x(x-5) < 0$$
$$\Rightarrow 0 < x < 5 \quad \text{—— (1)}$$

Also, $\log_{10}\left(\frac{5x-x^2}{4}\right) \geq 0$

$$\Rightarrow \frac{5x-x^2}{4} \geq 10^0 \Rightarrow 5x-x^2 \geq 4$$
$$\Rightarrow x^2-5x+4 \leq 0 \Rightarrow (x-4)(x-1) \leq 0$$
$$\Rightarrow 1 \leq x \leq 4 \quad \text{—— (2)}$$

from (1) and (2), domain is $[1, 4]$

2. The domain of the function

$$f(x) = \sqrt{x - \sqrt{1-x^2}}$$

Soln:

$$1-x^2 \geq 0 \Rightarrow x^2-1 \leq 0$$
$$\Rightarrow (x-1)(x+1) \leq 0$$
$$\Rightarrow -1 \leq x \leq 1 \quad \text{—— (1)}$$

Again, $x - \sqrt{1-x^2} \geq 0$

$$\Rightarrow \sqrt{1-x^2} \leq x$$
$$\Rightarrow 1-x^2 \leq x^2 \Rightarrow 2x^2-1 \geq 0$$
$$\Rightarrow x^2 - \frac{1}{2} \geq 0 \Rightarrow \left(x-\frac{1}{\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right) \geq 0$$
$$\Rightarrow -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}} \quad \text{—— (ii)}$$

But when $-\frac{1}{\sqrt{2}} \leq x \leq 0$, then

$x - \sqrt{1-x^2} < 0$ $\therefore$ domain $\left[\frac{1}{\sqrt{2}}, 1\right]$

3. Let $f:(-\infty,1]\to(-\infty,1]$ such that $f(x)=x(2-x)$ Then $f^{-1}(x)$ will be

Soln. $f(x)=x(2-x) \Rightarrow y=2x-x^2$

$$\Rightarrow x^2-2x+y=0$$

$$\Rightarrow x=\frac{2\pm\sqrt{4-4y}}{2}$$

$$=1\pm\sqrt{1-y}$$

But domain $x\in(-\infty,1]$

$\therefore x=1+\sqrt{1-y}$ is rejected

$$\therefore x=1-\sqrt{1-y}$$

i.e $f^{-1}(x)=1-\sqrt{1-x}$

4. The domain of the function $f(x)=\dfrac{1}{\sqrt{[x]^2-[x]-6}}$ is

Soln. $[x]^2-[x]-6>0$

$$\Rightarrow ([x]-3)([x]-2)>0$$

$$\Rightarrow [x]>3 \quad \text{or,} \quad [x]<2$$

$$\Rightarrow x\geq 4 \quad \text{or,} \quad x<2$$

$\therefore$ domain $(-\infty,2)\cup[4,\infty)$

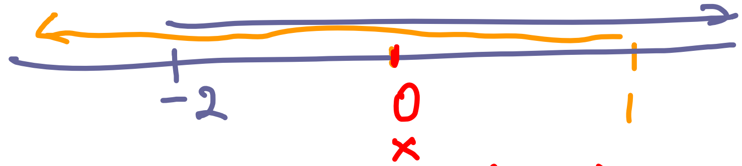
5. The domain of definition of the function $y=\dfrac{1}{\log(1-x)}+\sqrt{x+2}$ is

Soln:- $1-x>0 \Rightarrow x<1$ ——— (1)

$\log(1-x) \neq 0 \Rightarrow 1-x \neq 1 \Rightarrow x \neq 0$ —— (2)

$x+2 \geq 0 \Rightarrow x \geq -2$ —— (3)

from (1), (2) and (3)



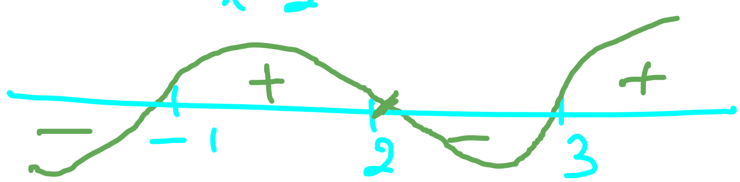
$\therefore x \in [-2, 0) \cup (0, 1)$

7. The domain of the function

$$f(x) = \sqrt{\frac{(x+1)(x-3)}{x-2}} \text{ is given by}$$

Soln.

$$\frac{(x+1)(x-3)}{x-2} \geq 0$$



$\therefore$ domain $x \in [-1, 2) \cup [3, \infty)$

6. The domain of the function

$$f(x) = \frac{1}{\sqrt{x^{12} - x^9 + x^4 - x + 1}}$$

Soln.

$$x^{12} - x^9 + x^4 - x + 1$$

$$= x^9(x^3-1) + x(x^3-1) + 1$$

$$= (x^9+x)(x^3-1) + 1$$

$$= x(x^8+1)(x-1)(x^2+x+1) + 1$$

$$= x(x-1)\underbrace{(x^8+1)}_{(+ve)}\underbrace{\left[\left(x+\frac{1}{2}\right)^2 + \frac{3}{4}\right]}_{+ve} + 1$$

when $x(x-1) \geq 0$,

i.e $x \leq 0$ or $x \geq 1$, then

$x^{12} - x^9 + x^4 - x + 1 \geq 0$

Now, when $0 < x < 1$, then

$x^{12} - x^9 + x^4 - x + 1$

$= (x^{12} + x^4) - (x^9 + x) + 1$

$= x^4(x^8+1) - x(x^8+1) + 1$

$\because 0 < x < 1$

$\therefore 0 < x^4 < x < 1$

$\Rightarrow 0 < x^4(x^8+1) < x(x^8+1) < 1$

$\Rightarrow 0 < x^4(x^8+1) - x(x^8+1) < 1$

$\therefore 1 + x^4(x^8+1) - x(x^8+1) > 0$

If means $x^{12} - x^9 + x^4 - x + 1 > 0$

for all $x \in R$

$\therefore$ domain $(-\infty, \infty)$

8. The domain of function

$f(x) = \sqrt{x-1} + \sqrt{5-x}$ is

Soln. $x - 1 \geq 0 \Rightarrow x \geq 1$ — (i)

$5 - x \geq 0 \Rightarrow x \leq 5$ — (ii)

from (i) and (ii) domain $[1, 5]$

9. The domain of the function

$$f(x) = \log_{10} \frac{x-5}{x^2-10x+24} - \sqrt[3]{x+5}$$

Soln: $\sqrt[3]{x+5}$ is defined for $x \in R$ — (1)

$$\frac{x-5}{x^2-10x+24} > 0$$

$$\Rightarrow \frac{x-5}{(x-4)(x-6)} > 0$$

— (ii)

$\therefore$ domain $x \in (4,5) \cup (6,\infty)$

10. The domain of the function

$$f(x) = \frac{1}{\sqrt{|x|-x}} \text{ is}$$

Soln: $|x| - x > 0 \Rightarrow x \in (-\infty, 0)$

11. The domain of the function

$$f(x) = \sqrt{\log_{10} \frac{3-x}{x}}$$

Soln: $\frac{3-x}{x} > 0 \Rightarrow \frac{x-3}{x} < 0$

$\Rightarrow 0 < x < 3$ — (1)

Also, $\log_{10}\left(\frac{3-x}{x}\right) \geq 0$

$\Rightarrow \frac{3-x}{x} \geq 10^0 \Rightarrow \frac{3-x}{x} \geq 1$

$\Rightarrow 1 - \frac{3-x}{x} \leq 0 \Rightarrow \frac{2x-3}{x} \leq 0$

$\Rightarrow \frac{x-3/2}{x} \leq 0 \Rightarrow 0 \leq x \leq \frac{3}{2}$

— (2)

from ① and ② $x \in (0, \frac{3}{2}]$

12. The domain of function

$$f(x) = \log_{10} |4-x^2| \text{ is}$$

Soln: $|4-x^2| > 0$ which is true for all x except $x = \pm 2$

$\therefore$ domain $R - \{-2, 2\}$

13. The domain of the function

$$f(x) = \sqrt{1-\sqrt{1-\sqrt{1-x^2}}} \text{ is}$$

Soln: $\sqrt{1-x^2} \geq 0 \Rightarrow x^2 - 1 \leq 0$

$\Rightarrow -1 \leq x \leq 1 \Rightarrow x \in [-1, 1]$

which is true for all functions

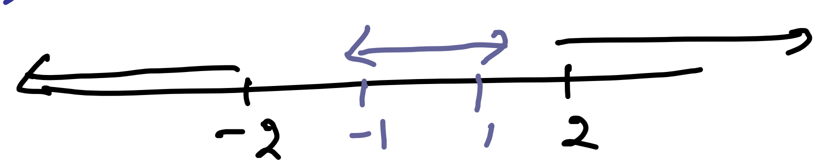
Hence, domain $x \in [-1, 1]$

14. The domain of the function

$$f(x) = \sqrt{\frac{1-|x|}{2-|x|}} \text{ is}$$

Soln: $\frac{1-|x|}{2-|x|} \geq 0 \Rightarrow \frac{|x|-1}{|x|-2} \geq 0$

$\Rightarrow |x| \geq 2$ or $|x| \leq 1$



$\therefore$ domain $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

15. The domain of the function $f(x) = \log_2 \log_3 \log_4 x$ is

Soln: $x > 0$ ——— (1)

$\log_4 x > 0 \Rightarrow x > 4^0 \Rightarrow x > 1$ — (2)

$\log_3 \log_4 x > 0 \Rightarrow \log_4 x > 3^0$

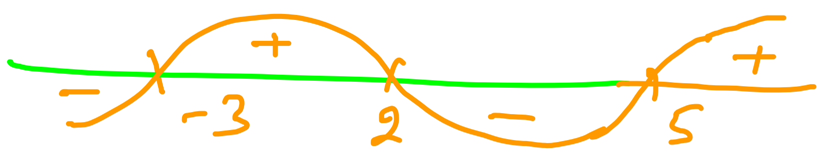
$\Rightarrow \log_4 x > 1 \Rightarrow x > 4$ — (3)

from (1), (2), (3) domain $(4, \infty)$

16. The domain of the function $f(x) = \sqrt{\dfrac{(x+3)}{(2-x)(x-5)}}$ is

Soln: $\dfrac{x+3}{(2-x)(x-5)} \geq 0$

$\Rightarrow \dfrac{x+3}{(x-2)(x-5)} \leq 0$



$\therefore$ domain $(-\infty, -3] \cup (2, 5)$

17. The domain of the function $f(x) = \sin^{-1}\left\{\log_2\left(\frac{1}{2}x^2\right)\right\}$

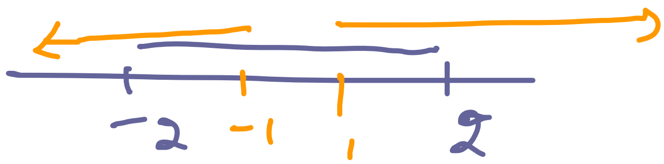
Soln: domain of $\sin^{-1} x$ is $[-1, 1]$

$\therefore -1 \leq \log_2\left(\frac{1}{2}x^2\right) \leq 1$ and $x \neq 0$

$$\Rightarrow \frac{1}{2} \le \frac{1}{2}x^2 \le 2$$

$$\Rightarrow 1 \le x^2 \le 4$$

$\Rightarrow$



$\therefore$ domain $x \in [-2,-1] \cup [1,2]$

18. The domain of the function

$$f(x) = \sqrt{3-x} + \cos^{-1}\left(\frac{3-2x}{5}\right) \text{ is}$$

Soln: $3 - x \ge 0 \Rightarrow x \le 3$ — (1)

$$-1 \le \frac{3-2x}{5} \le 1 \Rightarrow -5 \le 3-2x \le 5$$

$$\Rightarrow -8 \le -2x \le 2$$

$$\Rightarrow -1 \le x \le 4 \quad \text{— (ii)}$$

from (i) and (ii) $x \in [-1, 3]$

19. The domain of the function

$$f(x) = \sin^{-1}\left(\frac{1+x^3}{2x^{3/2}}\right) + \sqrt{\sin(\sin x)} + \log_{(3\{x\}+1)}(x^2+1) \text{ where}$$

$\{x\}$ denotes the fractional part of x is

Soln $\sin(\sin x) \ge 0 \Rightarrow x \in [2n\pi, (2n+1)\pi]$ — (1)

$$-1 \le \frac{1+x^3}{2x^{3/2}} \le 1$$

$$\Rightarrow -2 \le \frac{1+x^3}{x^{3/2}} \le 2$$

$$\Rightarrow -2 \le \frac{1}{x^{3/2}} + x^{3/2} \le 2$$

$$\Rightarrow x = -1, 1 \quad \text{—— (2)}$$

and $3\{x\}+1 \neq 1 \Rightarrow \{x\} \neq 0$

$$\Rightarrow x \notin I \quad \text{—— (3)}$$

$\therefore$ from (1), (2), (3) No value of x satisfy $\therefore$ domain is ϕ

20\. The domain of the function

$$f(x) = \frac{1}{1-x} + 2^{\sin^{-1}x} + \frac{1}{\sqrt{x-2}} \text{ is}$$

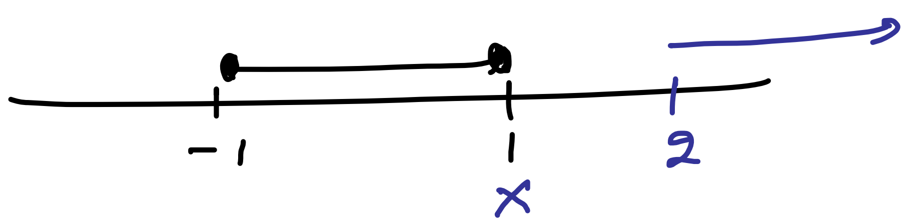
Sol^n:

$$1 - x \neq 0 \Rightarrow x \neq 1 \quad \text{—— (1)}$$

$$-1 \le x \le 1 \quad \text{—— (2)}$$

$$x - 2 > 0 \Rightarrow x > 2 \quad \text{—— (3)}$$

from (1), (2), (3), we have



no common solution satisfy eq^n (1), (2), (3) together. Hence domain is ϕ.