Explorer 46.

Q.1 If $\{x\}$ denotes the fractional part of x, then

$$\lim_{x\to 0^+} \frac{e^{\{x\}}-1}{x}$$

Soln:

$$\lim_{x\to 0^+} \frac{e^{x-[x]}-1}{x} = \lim_{x\to 0^+} \frac{e^{x-0}-1}{x}$$

$$\lim_{x\to 0^+} \frac{e^{x}-1}{x} = 1 \quad \text{Ans.}$$

Q.2 $$\lim_{x\to 0} \frac{\left(\sqrt{8xy-4x^2}+\sqrt{8xy}\right)^3}{x\sqrt{y^2-(x-y)^2}}$$

Soln: Taking x common

$$\lim_{x\to 0} \frac{\left[\sqrt{x}\left(\sqrt{8y-4x}+\sqrt{8y}\right)\right]^3}{x\sqrt{\cancel{y^2}-x^2-\cancel{y^2}+2xy}}$$

$$\lim_{x\to 0} \frac{\cancel{x\sqrt{x}}\left(\sqrt{8y-4x}+2\sqrt{2y}\right)^3}{\cancel{x\sqrt{x}}\sqrt{2y-x}}$$

$$= \frac{\left(2\sqrt{2}\sqrt{y}+2\sqrt{2}\sqrt{y}\right)^3}{\sqrt{2}\sqrt{y}} = \frac{128\sqrt{2}\,y\sqrt{y}}{\sqrt{2y}}$$

$$= 128y$$

Q.3. $\displaystyle \lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{1+k^2+k^4}$

Soln: $\because \dfrac{k}{1+k^2+k^4} = \dfrac{k}{1+2k^2+k^4-k^2}$

$$= \frac{k}{(1+k^2)^2-(k)^2} = \frac{k}{(1+k+k^2)(1-k+k^2)}$$

$$= \frac{1}{2}\left[\frac{1}{1-k+k^2} - \frac{1}{1+k+k^2}\right]$$

$$\therefore \lim_{n\to\infty} \frac{1}{2} \sum_{k=1}^{n} \left[\frac{1}{1-k+k^2} - \frac{1}{1+k+k^2}\right]$$

$$= \lim_{n\to\infty} \frac{1}{2}\left[\left(1-\cancel{\frac{1}{3}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{7}}\right)+\cdots \cdots + \left(\cancel{\frac{1}{1-n+n^2}} - \frac{1}{1+n+n^2}\right)\right]$$

$$= \lim_{n\to\infty} \frac{1}{2}\left[1 - \frac{1}{1+n+n^2}\right]$$

$$= \frac{1}{2}[1-0] = \frac{1}{2}$$

Q4 If $\displaystyle a = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{(r+2)\, r!}$

and $\displaystyle b = \lim_{x\to 0} \frac{e^{\sin x} - e^{x}}{\sin x - x}$ then

find the relation between a and b.

Soln:

$$a = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r+1}{(r+2)(r+1)\,r!}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{(r+2)-1}{(r+2)!}$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r+2}{(r+2)!} - \frac{1}{(r+2)!}\right)$$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{(r+1)!} - \frac{1}{(r+2)!}$$

$$= \lim_{n\to\infty} \left(\frac{1}{2!} - \cancel{\frac{1}{3!}}\right) + \left(\cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}}\right) + \cdots$$

$$\cdots + \left(\cancel{\frac{1}{(n+1)!}} - \frac{1}{(n+2)!}\right)$$

$$= \lim_{n\to\infty} \frac{1}{2} - \frac{1}{(n+2)!} = \frac{1}{2}\ \text{Ans}$$

when $n\to\infty$, $\frac{1}{(n+2)!} \to 0$

Now $b = \lim_{x\to 0} \frac{e^{\sin x} - e^{x}}{\sin x - x} \quad \left(\frac{0}{0}\right)$

$$= \lim_{x\to 0} \frac{e^{x}\left[e^{\sin x - x} - 1\right]}{\sin x - x}$$

$b = \lim_{x \to 0} e^x \times 1 = 1$

and $a = \frac{1}{2} \Rightarrow 2a = 1$

$(\because b = 1) \Rightarrow \boxed{2a = b}$

Ans.

Q.5 $\lim_{x \to \infty} \left(\sqrt{x^2 - x + 1} - ax - b\right) = 0$

Then for $k \geq 2$, $\lim_{n \to \infty} \sec^{2n}(k! \pi b)$

Soln:

$$\lim_{x \to \infty} \frac{\sqrt{x^2 - x + 1} - (ax+b) \times \sqrt{x^2 - x + 1} + (ax+b)}{\sqrt{x^2 - x + 1} + (ax+b)} = 0$$

$$= \lim_{x \to \infty} \frac{(x^2 - x + 1) - (ax+b)^2}{\sqrt{x^2 - x + 1} + (ax+b)} = 0$$

$$= \lim_{x \to \infty} \frac{(1 - a^2)x^2 - (1 + ab)x + 1 - b^2}{\sqrt{x^2 - x + 1} + (ax+b)} = 0$$

This is possible if degree of Dr is more than Nr.

$\Rightarrow 1 - a^2 = 0$, and $1 + ab = 0$

$\Rightarrow a = 1$, $b = -\frac{1}{a}$

i.e $b = 1$

Now, for $k \geq 2$,

$k!\,\pi b$ is integral multiple of π

$\therefore \sec(k!\,\pi b) = \pm 1$

$\Rightarrow \sec^2(k!\,\pi b) = 1$

$$\lim_{n \to \infty} \sec^{2n}(k!\,\pi b) = 1 = a$$

6. $$\lim_{n \to \infty} \lim_{x \to 0} \frac{1}{\left(1^{\cot^2 x} + 2^{\cot^2 x} + \dots + n^{\cot^2 x}\right)^{\tan^2 x}}$$

Soln:

$$\lim_{n \to \infty} \lim_{x \to 0} \frac{1}{n} \times \frac{1}{\left(\left(\frac{1}{n}\right)^{\cot^2 x} + \left(\frac{2}{n}\right)^{\cot^2 x} + \dots + 1\right)^{\tan^2 x}}$$

$= 0 \times$ finite value $= 0$

when $n \to \infty$, $\frac{1}{n} \to 0$

7. $$\lim_{x \to \infty} \frac{\log x^n - [x]}{[x]}, \ n \in N$$

Soln: $$\lim_{x \to \infty} \frac{n \log x - [x]}{[x]}$$

$$\lim_{x \to \infty} \frac{n \log x}{[x]} - 1 = 0 - 1$$

$= -1$

$\left(\because \lim_{x \to \infty} \frac{\log x}{[x]} = 0\right)$

8. If α = min. of (x^2+2x+3) and

$$\beta = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{(r+2)\,r!}$$

Then find $\sum_{r=0}^{n} \alpha^{r} \beta^{n-r}$

Soln. α = min of $[(x+1)^2+2] = 2$

$$\beta = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{(r+2)\,r!} = \frac{1}{2}$$

as it shown in Soln of Q.N. 4

Now, $\sum_{r=0}^{n} \alpha^{r} \beta^{n-r} = \sum_{r=0}^{n} \left(\frac{\alpha}{\beta}\right)^{r} \beta^{n}$

$$= \beta^{n} \sum_{r=0}^{n} (4)^{n}.$$

$$= \left(\frac{1}{2}\right)^{n} \left[1+4+4^2+ \cdots + 4^n\right]$$

$$= \frac{1}{2^n} \frac{4^{n+1}-1}{4-1} = \frac{4^{n+1}-1}{3\cdot 2^n} \text{ Ans}$$

Q. If $[x]$ denotes the integral part of x,

then $\lim_{x\to\infty} \frac{\log_e [x]}{x}$

Soln. When $x\to\infty$, $\frac{1}{x}\to 0$

$\therefore \lim_{x\to\infty} \frac{1}{x} \log_e [x] = 0 \times$ Finite value

$= 0$ Ans

Q.10 $\lim_{x\to 0}\left(1^{\operatorname{cosec}^2 x}+2^{\operatorname{cosec}^2 x}+\cdots+n^{\operatorname{cosec}^2 x}\right)^{\sin^2 x}$

Soln $\lim_{x\to 0}\left[\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 x}+\left(\frac{2}{n}\right)^{\operatorname{cosec}^2 x}+\cdots+1\right]^{\sin^2 x}\times n$

$\frac{1}{n}<1$, $\left(\frac{1}{n}\right)^{\operatorname{cosec}^2 x}\to\left(\frac{1}{n}\right)^{\infty}=1$

when $x\to 0$

$\sin^2 0$

$\therefore \lim_{x\to 0}\left[0+0+\cdots+1\right]\times n$

$=n$ Ans.

Q.N.11. $\lim_{x\to 1^+}\frac{\int_1^x |t-1|\,dt}{\sin(x-1)}$

Soln: This is in form of $\left(\frac{0}{0}\right)$

using L. Hospital Rule

$\lim_{x\to 1^+}\frac{|x-1|}{\cos(x-1)}=\frac{0}{1}=0$

Q.N 12 $\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$, $n\in N$

Soln: $\lim\quad y=\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$

taking log both the sides

$\log y=\frac{1}{n}\log\left(\frac{n!}{n^n}\right)$

$$= \frac{1}{n}\log\left(\frac{1\cdot 2\cdot 3\cdot \cdots \cdot n}{n^n}\right)$$

$$= \frac{1}{n}\log\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}\cdots\frac{n}{n}\right)$$

$$= \frac{1}{n}\left[\log\frac{1}{n} + \log\frac{2}{n} + \cdots + \log\frac{n}{n}\right]$$

$$= \frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

Now,

$$\lim_{n\to\infty}\log y = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\left(\frac{r}{n}\right)$$

$$= \int_0^1 \log x \, dx$$

$$= \int_0^1 \underset{\mathrm{I}}{\log x}\cdot \underset{\mathrm{II}}{1}\, dx$$

Integrating by parts,

$$\lim_{n\to\infty}\log y = \left[x\log x\right]_0^1 - \int_0^1 \frac{1}{\cancel{x}}\times\cancel{x}\, dx$$

$$= 0 - [x]_0^1 = -1$$

$$\lim_{n\to\infty}\log y = -1 \Rightarrow \lim_{n\to\infty} y = e^{-1}$$

$$\lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}} = \frac{1}{e} \quad \text{Ans}$$

Note: $\lim_{x\to 0} x\log x = \lim_{x\to 0}\frac{\log x}{\frac{1}{x}} \left(\frac{\infty}{\infty}\right)$

Using L Hospital Rule

$$= \lim_{x\to 0} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0} -x = 0$$

Q.13. If $a_r = \frac{x-a_r}{|x-a_r|}$, $r = 1, 2, 3 \cdots n$

and $a_1 < a_2 < a_3 \cdots < a_n$

Then $\lim_{x\to a_m} (a_1 a_2 a_3 \cdots a_n)$, $1 \le m \le n$

Sol^n:

Case i When $r < m \Rightarrow a_m - a_r > 0$

$\Rightarrow |a_m - a_r|$

$= a_m - a_r$

$$\lim_{x\to a_m} \frac{x-a_r}{|x-a_r|} = \frac{a_m - a_r}{a_m - a_r} = 1$$

Case ii when $r > m$

Then $a_m - a_r < 0$

$\Rightarrow |a_m - a_r| = -(a_m - a_r)$

$$\lim_{x\to a_m} \frac{x-a_r}{|x-a_r|} = (-1)^{n-m}$$

Case iii when $r = m$

$$\lim_{x\to a_m} \frac{x-a_r}{|x-a_r|} = \pm 1$$

i.e limit does not exists.

14.Q If $S_n = \sum_{r=1}^{n} a_r$ and $\lim_{n\to\infty} a_n = a$

then $\lim_{n\to\infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{r=1}^{n} r}}$

Soln:

$$S_n = a_1 + a_2 + \cdots + a_n$$

$$S_{n+1} = a_1 + a_2 + \cdots + a_n + a_{n+1}$$

$$\therefore S_{n+1} - S_n = a_{n+1}$$

$$\lim_{n\to\infty} \frac{S_{n+1} - S_n}{\sqrt{\sum_{r=1}^{n} r}}$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{\sqrt{\frac{n(n+1)}{2}}} = \lim_{n\to\infty} \frac{\sqrt{2}\, a_{n+1}}{n\sqrt{1+\frac{1}{n}}}$$

$$= \lim_{n\to\infty} \left(\frac{1}{n}\right) \frac{\sqrt{2}\, a}{\sqrt{1+\frac{1}{n}}} = 0$$

$\because \lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} a_{n+1} = a$

$$\frac{1}{n} \to 0$$

Q.15 $\lim_{n\to\infty} \dfrac{n^k \sin^2 n!}{n+1} = 0$

Soln: $\lim_{n\to\infty} n^{k-1} \dfrac{\sin^2 n!}{\left(1+\frac{1}{n}\right)} = 0$ (Divide by n)

This is possible if $\lim_{n\to\infty} n^{k-1}$ ∞

For which $0 \le k < 1$

Q.16 $\lim_{n\to\infty} 4^n \left(3^{\frac{1}{n}} - 1\right)$ $(\infty \times 0)$

Soln: $\lim_{n\to\infty} \dfrac{3^{\frac{1}{n}} - 1}{4^{-n}}$ $\left(\frac{0}{0}\right)$

using LH Rule

$\lim_{n\to\infty} \dfrac{3^{\frac{1}{n}} \log 3}{-4^{-n} \log 4}$

$= \lim_{n\to\infty} -4^n \, 3^{\frac{1}{n}} \dfrac{\log 3}{\log 4} = \infty$

17. $\lim_{h\to 0} \dfrac{\int_a^{x+h} \sin^4 t \, dt - \int_a^{x} \sin^4 t \, dt}{h}$

Soln: using L Hospital Rule

$\lim_{h\to 0} \dfrac{\sin^4 (x+h) - 0}{1} = \sin^4 x$

Ans.

18. $$\lim_{n\to\infty} \frac{3^3-1}{3^3+1}\cdot\frac{4^3-1}{4^3+1}\cdots\cdots\frac{n^3-1}{n^3+1}$$

Soln: $$\lim_{n\to\infty} \prod_{r=3}^{n} \frac{r^3-1}{r^3+1}$$

$$\lim_{n\to\infty} \prod_{r=3}^{n} \frac{(r-1)(r^2+r+1)}{(r+1)(r^2-r+1)}$$

$$\lim_{n\to\infty} \prod_{r=3}^{n} \left(\frac{r-1}{r+1}\right)\left(\frac{r^2+r+1}{r^2-r+1}\right)$$

$$\lim_{n\to\infty} \left(\frac{2}{\cancel{4}}\cdot\frac{3}{\cancel{5}}\cdot\frac{\cancel{4}}{\cancel{6}}\cdot\frac{\cancel{5}}{\cancel{7}}\cdots\frac{\cancel{n-3}}{\cancel{n-1}}\cdot\frac{\cancel{n-2}}{n}\cdot\frac{\cancel{n-1}}{n+1}\right)\cdot\left(\frac{\cancel{13}}{7}\cdot\frac{\cancel{21}}{\cancel{13}}\cdot\frac{\cancel{31}}{\cancel{21}}\cdots\frac{\cancel{n^2-n+1}}{\cancel{n^2-3n+3}}\cdot\frac{n^2+n+1}{\cancel{n^2-n+1}}\right)$$

$$\lim_{n\to\infty} \frac{2\cdot 3}{n(n+1)} \times \frac{n^2+n+1}{7}$$

$$\lim_{n\to\infty} \frac{6}{7}\,\frac{\cancel{n^2}\left(1+\frac{1}{n}+\frac{1}{n^2}\right)}{\cancel{n^2}\left(1+\frac{1}{n}\right)} = \frac{6}{7}$$

when $n\to\infty$, $\frac{1}{n}\to 0$, $\frac{1}{n^2}\to 0$

Q. $$\lim_{x\to 2} \frac{(\cos\alpha)^x+(\sin\alpha)^x-1}{x-2}$$

Soln: This is in form of $\frac{0}{0}$

Applying L Hospital Rule

$$\lim_{x \to 2} \frac{(\cos\alpha)^x \log(\cos\alpha) + (\sin\alpha)^x \log(\sin\alpha)}{1}$$

$$= \cos^2\alpha \log\cos\alpha + \sin^2\alpha \log(\sin\alpha)$$

Q. For all real a, $\lim_{n \to \infty} \frac{a^n}{n!}$.

Soln Case I :- when $|a| < 1$

when $n \to \infty$ $a^n \to 0$

$\therefore \lim_{n \to \infty} \frac{a^n}{n!} = 0$

Case II when $|a| \geq 1$

when $n \to \infty$, $a^n \to \infty$

Then $\lim_{n \to \infty} \frac{a^n}{n!}$ $\left(\frac{\infty}{\infty}\right)$ but $a < n$

$\therefore \lim_{n \to \infty} \frac{a}{n} \frac{a}{n-1} \cdots \frac{a}{1} = 0$

when $n \to \infty$, $\frac{1}{n} \to 0$

21. $\lim_{x \to \frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3} - x\right)}{2\cos x - 1}$

Soln: This is in form of $\left(\frac{0}{0}\right)$

using L Hospital Rule,

$\therefore$

$$\lim_{x \to \frac{\pi}{3}} \frac{\cos\left(\frac{\pi}{3} - x\right)(-1)}{-2\sin x} = \frac{1}{2 \times \frac{\sqrt{3}}{2}}$$

$$= \frac{1}{\sqrt{3}} \text{ Ans.}$$

Q. If $0 < x < y$, then $\lim_{n \to \infty} \left(y^n + x^n\right)^{\frac{1}{n}}$

Soln. $\lim_{n \to \infty} \left[y^n \left(1 + \left(\frac{x}{y}\right)^n\right)\right]^{\frac{1}{n}}$

$$\lim_{n \to \infty} \left(y^n\right)^{\frac{1}{n}} \left[1 + \left(\frac{x}{y}\right)^n\right]^{\frac{1}{n}}$$

$$\lim_{n \to \infty} y \left[1 + \left(\frac{x}{y}\right)^n\right]^{\frac{1}{n}}$$

$$\lim_{n \to \infty} y \, (1 + 0)^0 = y \text{ Ans}$$

$\because x < y \Rightarrow \frac{x}{y} < 1$

when $n \to \infty$, $\left(\frac{x}{y}\right)^n \to 0$

23. $\lim_{x \to \infty} \frac{x(\log x)^3}{1 + x + x^2}$

Soln. This is in form of $\left(\frac{\infty}{\infty}\right)$

Applying L Hospital Rule

$$\lim_{x \to \infty} \frac{(\log x)^3 + 3(\log x)^2}{2x + 1}$$

$$\lim_{x \to \infty} \frac{3(\log x)^2 \times \frac{1}{x} + 6 \log x \times \frac{1}{x}}{2}$$

$$\lim_{x\to\infty} \frac{3(\log x)^2 + 6\log x}{2x}$$

Apply again L H Rule

$$\lim_{x\to\infty} \frac{3\times 2\log x \times \frac{1}{x} + 6\times\frac{1}{x}}{2}$$

$$\lim_{x\to\infty} \frac{6\log x + 6}{2x}.$$

$$\lim_{x\to\infty} \frac{6\times \frac{1}{x} + 0}{2} = \lim_{x\to\infty} \frac{3}{x} = 0$$

24Q. $\lim_{n\to\infty} \cos\frac{x}{2}\cos\frac{x}{4}\cos\frac{x}{8}\cdots\cos\frac{x}{2^n}$

Soln $\lim_{n\to\infty} \frac{1}{2^n\sin\frac{x}{2^n}} \sin x$

$$\lim_{n\to\infty} \frac{\sin x}{x \times \left(\frac{\sin\frac{x}{2^n}}{\frac{x}{2^n}}\right)} = \frac{\sin x}{x}$$

Ans.

$$\left[\because \lim_{n\to\infty} \frac{\sin\frac{x}{2^n}}{\frac{x}{2^n}} = 1\right]$$

Q.25. $\lim_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$

Soln. using L Hospital Rule

$$\lim_{y\to 0} \frac{(x+y)\sec(x+y)\tan(x+y) + \sec(x+y) - 0}{1}$$

$= x \sec x \tan x + \sec x$ Ans

$= \sec x\,(x\tan x + 1)$

26. $\lim_{x \to 0} \dfrac{1 - \cos x\sqrt{\cos 2x}}{x^2}$

Sol^n: $\lim_{x \to 0} \dfrac{1 - \cos x\sqrt{\cos 2x}}{x^2} \times \dfrac{1 + \cos x\sqrt{\cos 2x}}{1 + \cos x\sqrt{\cos 2x}}$

$\lim_{x \to 0} \dfrac{1 - \cos^2 x \cos 2x}{x^2\left[1 + \cos x\sqrt{\cos 2x}\right]}$

$\lim_{x \to 0} \dfrac{1 - \cos^2 x\,(1 - 2\sin^2 x)}{x^2\left[1 + \cos x\sqrt{\cos 2x}\right]}$

$\lim_{x \to 0} \dfrac{\sin^2 x + 2\sin^2 x \cos^2 x}{x^2\left[1 + \cos x\sqrt{\cos 2x}\right]}$

$\lim_{x \to 0} \left(\dfrac{\sin x}{x}\right)^2 \left[\dfrac{1 + 2\cos^2 x}{1 + \cos x\sqrt{\cos 2x}}\right] = (1)^2 \times \dfrac{3}{2}$

$= \dfrac{3}{2}$ Ans.

27. $\lim_{x \to 0} \dfrac{\tan^{-1} x - \sin^{-1} x}{x^3}$

Sol^n using expansion.

$$\lim_{x \to 0} \frac{\left(\cancel{x} - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right) - \left(\cancel{x} + \frac{x^3}{6} + \frac{3}{40}x^5 + \cdots\right)}{x^3}$$

$\lim_{x \to 0} \left(-\dfrac{1}{3} - \dfrac{1}{6}\right) + \left(\dfrac{1}{5} - \dfrac{3}{40}\right)x^2 + \cdots$

$= -\dfrac{1}{3} - \dfrac{1}{6} = -\dfrac{1}{2}$ Ans

Q. 28 $\lim\limits_{x\to 0} \dfrac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2}$

Soln: $\lim\limits_{x\to 0} \dfrac{e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 - \dots\right] - e + \frac{ex}{2}}{x^2}$

$\lim\limits_{x\to 0} \dfrac{\left[\cancel{e} - \cancel{\frac{ex}{2}} + \frac{11ex^2}{24} \dots\right] - \cancel{e} + \cancel{\frac{ex}{2}}}{x^2}$

$= \dfrac{11e}{24}$ Ans.

Q29. $\lim\limits_{h\to 0} \dfrac{(a+h)^2 \sin(a+h) - a^2 \sin a}{h}$

Soln Applying L Hospital Rule

$\lim\limits_{h\to 0} \dfrac{(a+h)^2 \cos(a+h) + 2(a+h)\sin(a+h) - 0}{1}$

$= a^2 \cos a + 2a \sin a$ Ans.

28. $\lim\limits_{x\to \infty} \sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}$

Soln $\lim\limits_{x\to\infty} \sqrt{x + \sqrt{x+\sqrt{x}}} - \sqrt{x} \times \dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$

$\lim\limits_{x\to\infty} \dfrac{\cancel{x} + \sqrt{x+\sqrt{x}} - \cancel{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}} + \sqrt{x}}$

$\lim\limits_{x\to\infty} \dfrac{\cancel{\sqrt{x}}\left(\sqrt{1+\sqrt{\frac{1}{x}}}\right)}{\cancel{\sqrt{x}}\left(\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1\right)} = \dfrac{1}{2}$