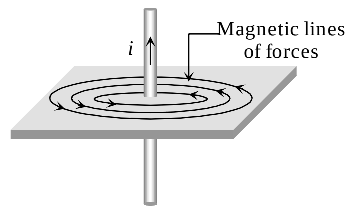
Magnetic Effect of Current



Oersted found that a magnetic field is established around a current carrying conductor.

Magnetic field exists as long as there is current in the wire.

The direction of magnetic field was found to be changed when direction of current was reversed.

Note : ❐ A moving charge produces magnetic as well as electric field, unlike a stationary charge which only produces electric field.

Biot Savart's Law

Biot-Savart's law is used to determine the magnetic field at any point due to a current carrying conductors.

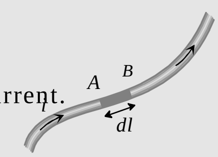
This law is although for infinitesimally small conductors yet it can be used for long conductors. In order to understand the Biot-Savart's law, we need to understand the term current-element.

Current element

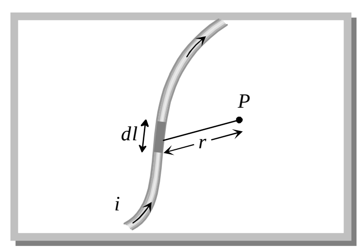
It is the product of current and length of infinitesimal segment of current carrying wire.

The current element is taken as a vector quantity. Its direction is same as the direction of current.

Current element $AB = i\vec{dl}$



In the figure shown below, there is a segment of current carrying wire and P is a point where magnetic field is to be calculated. $i\vec{dl}$ is a current element and r is the distance of the point 'P' with respect to the current element $i\vec{dl}$. According to Biot-Savart Law, magnetic field at point 'P' due to the current element $i\vec{dl}$ is given by the expression, $\boldsymbol{dB = k\frac{i\,dl\sin\theta}{r^2}}$ also $B = \int dB = \frac{\mu_0 i}{4\pi}.\int\frac{dl\sin\theta}{r^2}$



In C.G.S. : $k = 1 \Rightarrow dB = \frac{idl\sin\theta}{r^2}$ *Gauss*

In S.I. : $k = \frac{\mu_0}{4\pi} \Rightarrow dB = \frac{\mu_0}{4\pi}\cdot\frac{idl\sin\theta}{r^2}$ *Tesla*

where μ_0 = Absolute permeability of air or vacuum $= 4\pi\times10^{-7}\frac{Wb}{Amp-metre}$. It's other units are $\frac{Henry}{metre}$ or $\frac{N}{Amp^2}$ or $\frac{Tesla-metre}{Ampere}$

(1) Different forms of Biot-Savarts law

Vector form	Biot-Savarts law in terms of current density	Biot-savarts law in terms of charge and it's velocity
Vectorially, $d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \hat{r})}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i(d\vec{l} \times \vec{r})}{r^3}$ $\Rightarrow$ Direction of $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$. This is given by right hand screw rule.	In terms of current density $d\vec{B} = \frac{\mu_0}{4\pi} \frac{\vec{J} \times \vec{r}}{r^3} dV$ where $j = \frac{i}{A} = \frac{idl}{Adl} = \frac{idl}{dV}$ = current density at any point of the element, dV = volume of element	In terms of charge and it's velocity, $d\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v} \times \vec{r})}{r^3}$ $\because\ id\vec{l} = \frac{q}{dt} d\vec{l} = q\frac{d\vec{l}}{dt} = q\vec{v}$

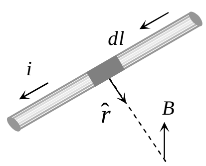
(2) Similarities and differences between Biot-Savart law and Coulomb's Law

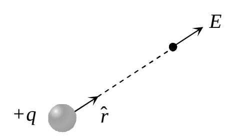
(i) The current element produces a magnetic field, whereas a point charge produces an electric field.

(ii) The magnitude of magnetic field varies as the inverse square of the distance from the current element, as does the electric field due to a point charge.

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{l} \times \hat{r}}{r^2} \quad \text{Biot-Savart Law} \qquad \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \quad \text{Coulomb's Law}$$

(iii) The electric field created by a point charge is radial, but the magnetic field created by a current element is perpendicular to both the length element $d\vec{l}$ and the unit vector $\hat{r}$.

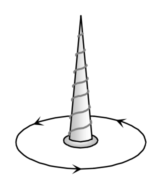




Direction of Magnetic Field

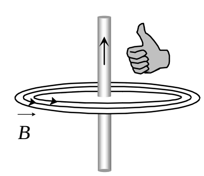
The direction of magnetic field is determined with the help of the following simple laws :

(1) Maxwell's cork screw rule



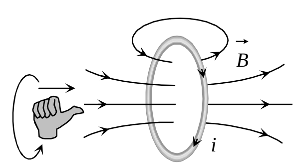
According to this rule, if we imagine a right handed screw placed along the current carrying linear conductor, be rotated such that the screw moves in the direction of flow of current, then the direction of rotation of the thumb gives the direction of magnetic lines of force.

(2) Right hand thumb rule



According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.

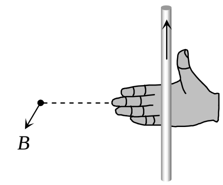
(3) Right hand thumb rule of circular currents



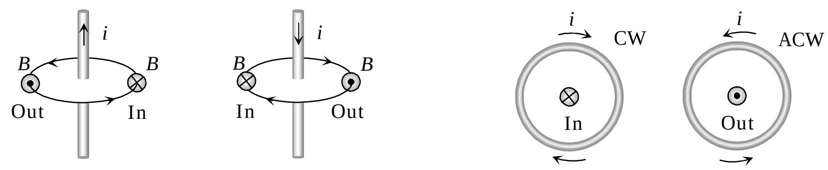
According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.

(4) Right hand palm rule

If we stretch our right hand such that fingers point towards the point. At which magnetic field is required while thumb is in the direction of current then normal to the palm will show the direction of magnetic field.



Note : ❐ If magnetic field is directed perpendicular and into the plane of the paper it is represented by ⊗ (cross) while if magnetic field is directed perpendicular and out of the plane of the paper it is represented by ⊙ (dot)



In : Magnetic field is away from the observer or perpendicular inwards.

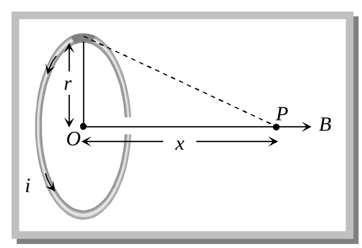
Out : Magnetic field is towards the observer or perpendicular outwards.

Application of Biot-Savarts Law

(1) Magnetic field due to a circular current

If a coil of radius r, carrying current i then magnetic field on it's axis at a distance x from its centre given by

$$\boldsymbol{B_{axis}} = \frac{\boldsymbol{\mu_0}}{\boldsymbol{4\pi}} \cdot \frac{\boldsymbol{2\pi N i r^2}}{\boldsymbol{(x^2 + r^2)^{3/2}}};$$ where N = number of turns in coil.



Different cases

Case 1 : Magnetic field at the centre of the coil

(i) At centre $x = 0 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i}{r} = \frac{\mu_0 N i}{2r} = B_{\max}$

(ii) For single turn coil $N = 1 \Rightarrow B_{centre} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r} = \frac{\mu_0 i}{2r}$ (iii) In C.G.S. $\frac{\mu_0}{4\pi} = 1 \Rightarrow B_{centre} = \frac{2\pi i}{r}$

Note : ❐ $B_{centre} \propto N$ (i, r constant), $B_{centre} \propto i$ (N, r constant), $B_{centre} \propto \frac{1}{r}$ (N, i constant)

Case 2 : Ratio of B_{centre} and B_{axis}

The ratio of magnetic field at the centre of circular coil and on it's axis is given by $\boldsymbol{\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}}$

(i) If $x = \pm a,\ B_c = 2\sqrt{2}\, B_a$ $x = \pm \frac{a}{2},\ B_c = \frac{5\sqrt{5}}{8} B_a$ $x = \pm \frac{a}{\sqrt{2}},\ B_c = \left(\frac{3}{2}\right)^{3/2} B_a$

(ii) If $B_a = \frac{B_c}{n}$ then $x == \pm r\sqrt{(n^{2/3} - 1)}$ and if $B_a = \frac{B_c}{\sqrt{n}}$ then $x == \pm r\sqrt{(n^{1/3} - 1)}$

Case 3 : Magnetic field at very large/very small distance from the centre

(i) If $x >> r$ (very large distance) $\Rightarrow B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{x^3} = \frac{\mu_0}{4\pi} \cdot \frac{2NiA}{x^3}$ where $A = \pi r^2$ = Area of each turn of the coil.

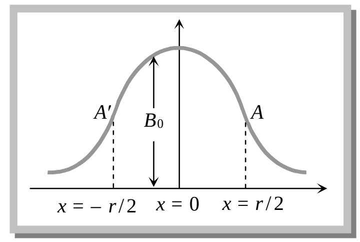
(ii) If $x << r$ (very small distance) $\Rightarrow$ $B_{axis} \neq B_{centre}$, but by using binomial theorem and neglecting higher power of $\frac{x^2}{r^2}$; $B_{axis} = B_{centre}\left(1 - \frac{3}{2}\frac{x^2}{r^2}\right)$

Case 4 : *B*-*x* curve

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.

B varies non-linearly with distance x as shown in figure and is maximum when $x^2 = \min = 0$, *i.e.*, the point is at the centre of the coil and it is zero at $x = \pm\infty$.

Point of inflection (A and A') : Also known as points of curvature change or pints of zero curvature.



(i) At these points B varies linearly with $x \Rightarrow \frac{dB}{dx} =$ constant $\Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) They locates at $x = \pm\frac{r}{2}$ from the centre of the coil.

(iii) Separation between point of inflextion is equal to radius of coil (r)

(iv) Application of points of inflextion is "Hamholtz coils" arrangement.

Note : ❒ The magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}\,r}$

(2) Helmholtz coils

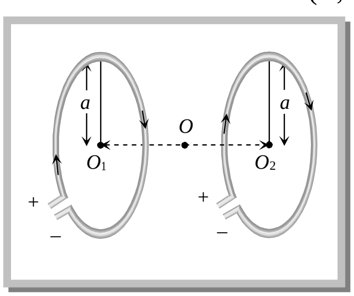
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

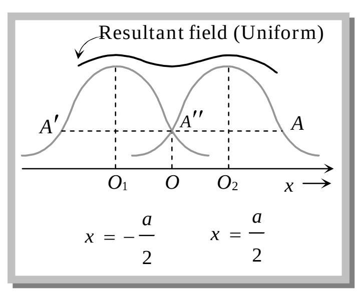
(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\,\frac{\mu_0 Ni}{R} = 1.432\,B$, where $B = \frac{\mu_0 Ni}{2R}$

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

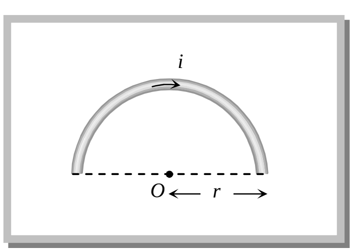
(v) Number of points of inflextion $\Rightarrow$ Three (A, A', A'')

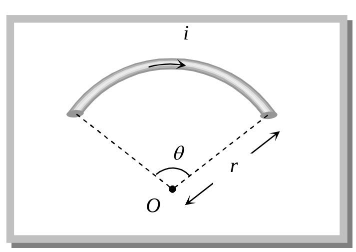


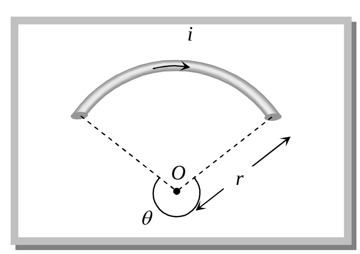


Note : ❒ The device whose working principle based on this arrangement and in which uniform magnetic field is used called as "Halmholtz galvanometer".

(3) Magnetic field due to current carrying circular arc : Magnetic field at centre O







$$B = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} = \frac{\mu_0 i}{4r} \qquad B = \frac{\mu_0}{4\pi}.\frac{\theta i}{r} \qquad B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r}$$

Special results

If magnetic field at the centre of circular coil is denoted by $B_0 \left(= \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} \right)$

Magnetic field at the centre of arc which is making an angle θ at the centre is

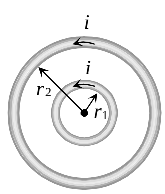
$$B_{arc} = \left(\frac{B_0}{2\pi} \right).\theta$$

Angle at centre	Magnetic field at centre in term of B_0
360^o (2π)	B_0
180^o (π)	B_0 / 2
120^o $(2\pi/3)$	B_0 / 3
90^o $(\pi/2)$	B_0 / 4
60^o $(\pi/3)$	B_0 / 6
30^o $(\pi/6)$	B_0 / 12

(4) Concentric circular loops (N = 1)

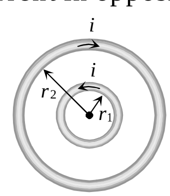
(i) Coplanar and concentric : It means both coils are in same plane with common centre

(a) Current in same direction



$$B_1 = \frac{\mu_0}{4\pi} 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2} \right)$$

(b) Current in opposite direction



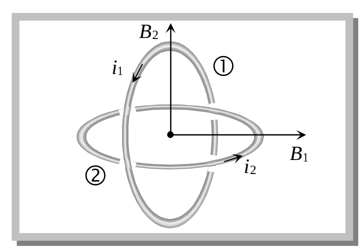
$$B_2 = \frac{\mu_0}{4\pi} 2\pi i \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

Note : ❒ $\frac{B_1}{B_2} = \left(\frac{r_2 + r_1}{r_2 - r_1} \right)$

(ii) Non-coplanar and concentric : Plane of both coils are perpendicular to each other

Magnetic field at common centre

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



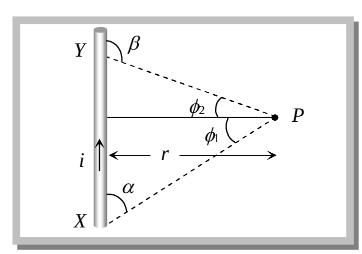
(5) Magnetic field due to a straight current carrying wire

Magnetic field due to a current carrying wire at a point P which lies at a perpendicular distance r from the wire as shown is given as

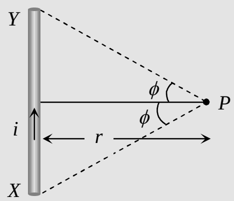
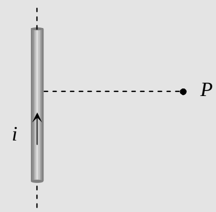
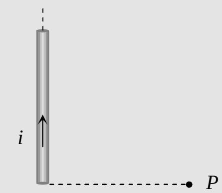
$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

From figure $\alpha = (90^o - \phi_1)$ and $\beta = (90^o + \phi_2)$

Hence $B = \frac{\mu_o}{4\pi}.\frac{i}{r}(\cos\alpha - \cos\beta)$

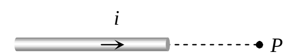


Different cases

Case 1 : When the linear conductor XY is of finite length and the point P lies on it's perpendicular bisector as shown	Case 2 : When the linear conductor XY is of infinite length and the point P lies near the centre of the conductor	Case 3 : When the linear conductor is of infinite length and the point P lies near the end Y or X
 $\phi_1 = \phi_2 = \phi$ So $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(2\sin\phi)$	 $\phi_1 = \phi_2 = 90^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 90^o] = \frac{\mu_0}{4\pi}\frac{2i}{r}$	 $\phi_1 = 90^o$ and $\phi_2 = 0^o$. So, $B = \frac{\mu_0}{4\pi}\frac{i}{r}[\sin 90^o + \sin 0^o] = \frac{\mu_0}{4\pi}\frac{i}{r}$

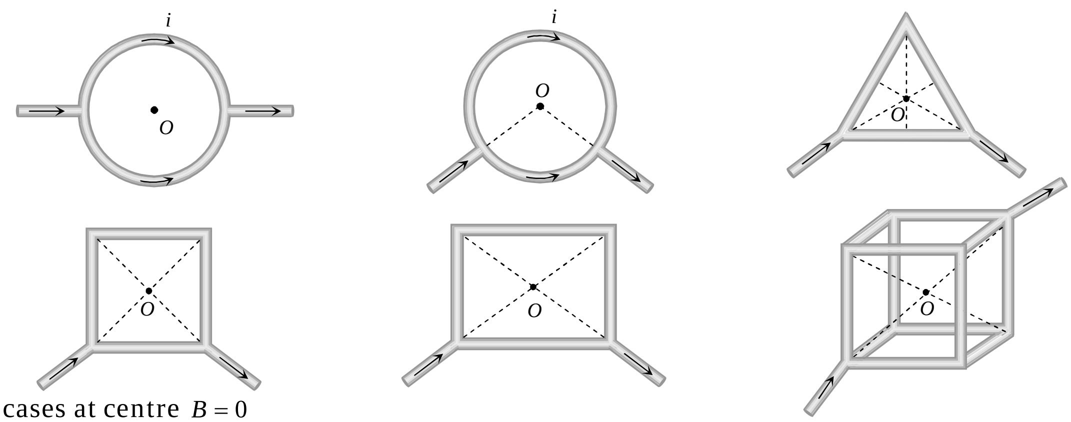
Note : ❒ When point P lies on axial position of current carrying conductor then magnetic field at P

$B = 0$



❒ The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.

(6) Zero magnetic field : If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.



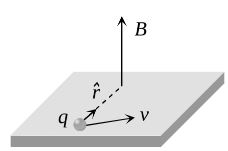
In all cases at centre $B = 0$

Concepts

☞ *If a current carrying circular loop (n = 1) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field i.e. $B_{(n\ turn)} = n^2 B_{(single\ turn)}$*

☞ *When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in East-West direction.*

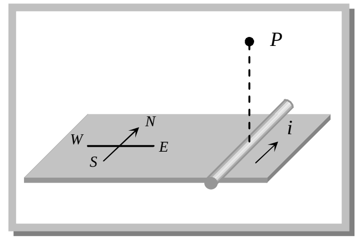
☞ *Magnetic field ($\vec{B}$) produced by a moving charge q is given by $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\hat{r})}{r^2}$; where v = velocity of charge and v << c (speed of light).*



☞ *If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path* $B = \frac{\mu_0}{4\pi}.\frac{ev}{r^2}$.

Example

Example: 1 Current flows due north in a horizontal transmission line. Magnetic field at a point P vertically above it directed



(a) North wards

(b) South wards

(c) Toward east

(d) Towards west

Solution : (c) By using right hand thumb rule or any other rule which helps to determine the direction of magnetic field.

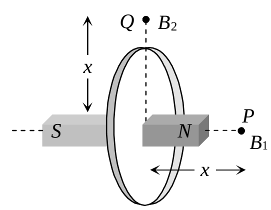
Example: 2 Magnetic field due to a current carrying loop or a coil at a distant axial point P is B_1 and at an equal distance in it's plane is B_2 then $\frac{B_1}{B_2}$ is

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) None of these

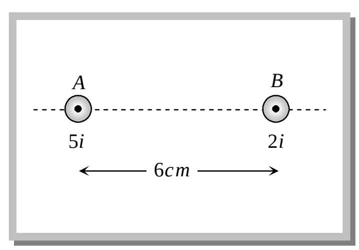
Solution : (a) Current carrying coil behaves as a bar magnet as shown in figure.

We also knows for a bar magnet, if axial and equatorial distance are same then $B_a = 2B_e$

Hence, in this equation $\frac{B_1}{B_2} = \frac{2}{1}$



Example: 3 Find the position of point from wire 'B' where net magnetic field is zero due to following current distribution



(a) 4 *cm*

(b) $\frac{30}{7}$ *cm*

(c) $\frac{12}{7}$ *cm*

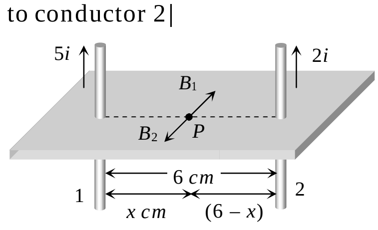
(d) 2 *cm*

Solution : (c) Suppose P is the point between the conductors where net magnetic field is zero.

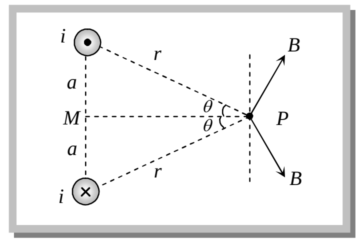
So at P |Magnetic field due to conductor 1| = |Magnetic field due to conductor 2|

i.e. $\frac{\mu_0}{4\pi}.\frac{2(5i)}{i} = \frac{\mu_0}{4\pi}.\frac{2(2i)}{(6-x)} \Rightarrow \frac{5}{x} = \frac{9}{6-x} \Rightarrow x = \frac{30}{7}cm$

Hence position from $B = 6 - \frac{30}{7} = \frac{12}{7}cm$



Example: 4 Find out the magnitude of the magnetic field at point P due to following current distribution



(a) $\frac{\mu_0 ia}{\pi r^2}$

(b) $\frac{\mu_0 ia^2}{\pi r}$

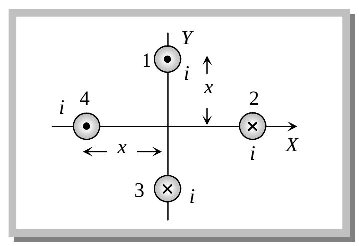
(c) $\frac{\mu_0 ia}{2\pi r^2}$

(d) $\frac{2\mu_0 ia}{\pi r^2}$

Solution : (a) Net magnetic field at P, $B_{net} = 2B\sin\theta$; where B = magnetic field due to one wire at $P = \frac{\mu_0}{4\pi}.\frac{2i}{r}$

and $\sin\theta = \frac{a}{r}$ $\quad \therefore \; B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{2i}{r}\times\frac{a}{r} = \frac{\mu_0 ia}{\pi r^2}$.

Example: 5 What will be the resultant magnetic field at origin due to four infinite length wires. If each wire produces magnetic field 'B' at origin



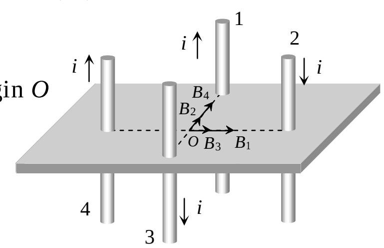
(a) $4\,B$

(b) $\sqrt{2}\,B$

(c) $2\sqrt{2}\,B$

(d) Zero

Solution : (c) Direction of magnetic field (B_1, B_2, B_3 and B_4) at origin due to wires 1, 2, 3 and 4 are shown in the following figure.



$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0}{4\pi}.\frac{2i}{x} = B$. So net magnetic field at origin O

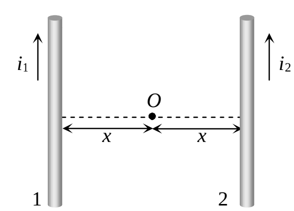
$$B_{net} = \sqrt{(B_1 + B_2)^2 + (B_2 + B_4)^2}$$

$$= \sqrt{(2B)^2 + (2B)^2} = 2\sqrt{2}B$$

Example: 6 Two parallel, long wires carry currents i_1 and i_2 with $i_1 > i_2$. When the currents are in the same direction, the magnetic field at a point midway between the wires is 10 μT. If the direction of i_2 is reversed, the field becomes 30 μT. The ratio i_1 / i_2 is

(a) 4 (b) 3 (c) 2 (d) 1

Solution : (c) Initially when wires carry currents in the same direction as shown.



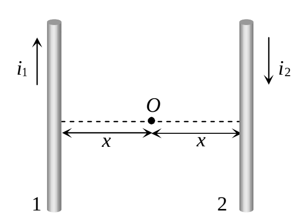
Magnetic field at mid point O due to wires 1 and 2 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \odot$$

Hence net magnetic field at O $\quad B_{net} = \frac{\mu_0}{4\pi}\times\frac{2}{x}(i_1 - i_2)$

$$\Rightarrow \; 10\times10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 - i_2) \qquad .....(i)$$

If the direction of i_2 is reversed then



$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1}{x} \otimes \text{ and } B_2 = \frac{\mu_0}{4\pi}.\frac{2i_2}{x} \otimes$$

So $B_{net} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2) \;\Rightarrow\; 30\times10^{-6} = \frac{\mu_0}{4\pi}.\frac{2}{x}(i_1 + i_2)$ ......(ii)

Dividing equation (ii) by (i) $\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{1} \Rightarrow \frac{i_1}{i_2} = \frac{2}{1}$

Example: 7 A wire of fixed length is turned to form a coil of one turn. It is again turned to form a coil of three turns. If in both cases same amount of current is passed, then the ratio of the intensities of magnetic field produced at the centre of a coil will be

(a) 9 times of first case (b) $\frac{1}{9}$ times of first case (c) 3 times of first case (d) $\frac{1}{3}$ times of first case

Solution : (a) Magnetic field at the centre of n turn coil carrying current i $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi n i}{r}$ ......(i)

For single turn $n = 1$ $B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}$ .....(ii)

If the same wire is turn again to form a coil of three turns *i.e.* $n = 3$ and radius of each turn $r' = \frac{r}{3}$

So new magnetic field at centre $B' = \frac{\mu_0}{4\pi} \cdot \frac{2\pi(3)}{r'}$ $\Rightarrow B' = 9 \times \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}$ ......(iii)

Comparing equation (ii) and (iii) gives $B' = 9B$.

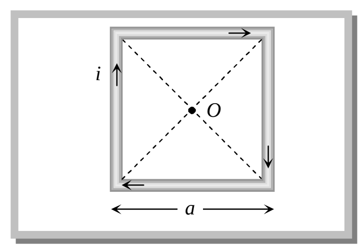
Example: 8 A wire in the form of a square of side a carries a current i. Then the magnetic induction at the centre of the square wire is (Magnetic permeability of free space = μ_0)

(a) $\frac{\mu_0 i}{2\pi a}$

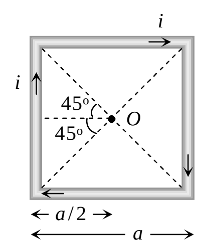
(b) $\frac{\mu_0 i\sqrt{2}}{\pi a}$

(c) $\frac{2\sqrt{2}\mu_0 i}{\pi a}$

(d) $\frac{\mu_0 i}{\sqrt{2}\pi a}$



Solution : (c) Magnetic field due to one side of the square at centre O



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2i \sin 45^o}{a/2}$$

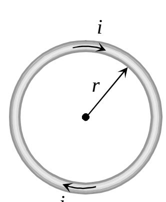
$$\Rightarrow B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\sqrt{2}\, i}{a}$$

Hence magnetic field at centre due to all side $B_{net} = 4B_1 = \frac{\mu_0 (2\sqrt{2}\, i)}{\pi a}$

Example: 9 The ratio of the magnetic field at the centre of a current carrying circular wire and the magnetic field at the centre of a square coil made from the same length of wire will be

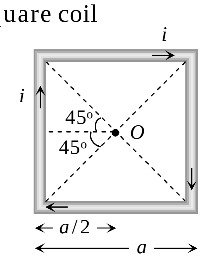
(a) $\frac{\pi^2}{4\sqrt{2}}$ (b) $\frac{\pi^2}{8\sqrt{2}}$ (c) $\frac{\pi}{2\sqrt{2}}$ (d) $\frac{\pi}{4\sqrt{2}}$

Solution : (b) Circular coil



Length $L = 2\pi r$

Square coil

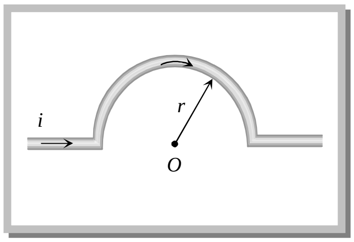


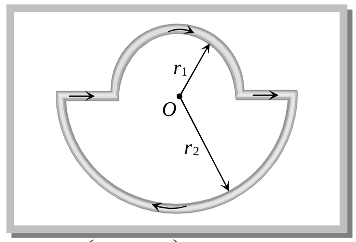
Length $L = 4a$

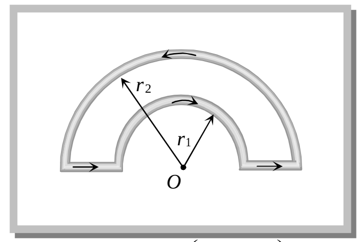
Magnetic field $B = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = \frac{\mu_0}{4\pi}.\frac{4\pi^2 i}{r}$ $\qquad$ $B = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\, i}{a}$ $B = \frac{\mu_0}{4\pi}.\frac{8\sqrt{2}\, i}{a}$

Hence $\frac{B_{circular}}{B_{square}} = \frac{\pi^2}{8\sqrt{2}}$

Example: 10 Find magnetic field at centre O in each of the following figure

(i) 

(ii) 

(iii) 

(i)

(a) $\frac{\mu_0 i}{r}\otimes$

(b) $\frac{\mu_0 i}{2r}\odot$

(c) $\frac{\mu_0 i}{4r}\otimes$

(d) $\frac{\mu_0 i}{4r}\odot$

(ii)

(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$

(d) Zero

(iii)

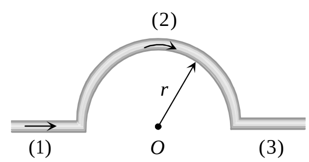
(a) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$

(b) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(c) $\frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\odot$

(d) Zero

Solution : (i) (c) Magnetic field at O due to parts 1 and 3, $B_1 = B_3 = 0$

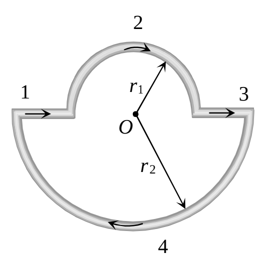


While due to part (2) $\quad B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes$

$\therefore$ Net magnetic field at centre O,

$$B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \Rightarrow B_{net} = \frac{\mu_0 i}{4r}\otimes$$

(ii) (b) $B_1 = B_3 = 0$

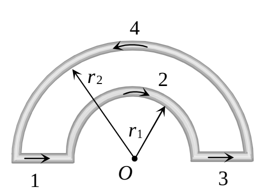


$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

$$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$$

So $B_{net} = B_2 + B_4 = \frac{\mu_0}{4\pi}.\pi i\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\otimes$

(iii) (a) $B_1 = B_3 = 0$



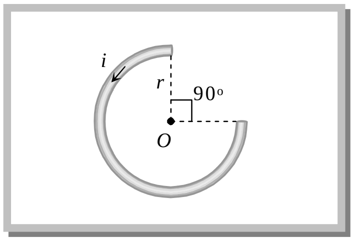
$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_1}\otimes$$

$B_4 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r_2}\otimes$ $\qquad$ As $|B_2| > |B_4|$

So $B_{net} = B_2 - B_4 \Rightarrow B_{net} = \frac{\mu_0 i}{4}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\otimes$

Example: 11 Find magnetic field at centre O in each of each of the following figure

(i) $\qquad$ (ii) $\qquad$ (iii)

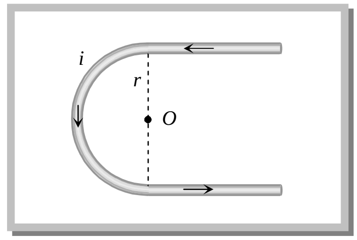


(a) $\frac{\mu_0 i}{2r} \odot$

(b) $\frac{\mu_0 i}{2r} \otimes$

(c) $\frac{3\mu_0 i}{8r} \otimes$

(d) $\frac{3\mu_0 i}{8r} \odot$

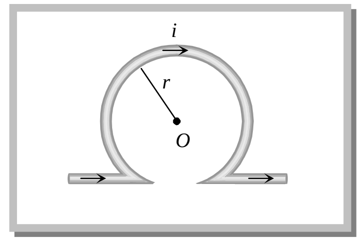


(a) $\frac{\mu_0}{2\pi}\frac{i}{r}(\pi - 2) \otimes$

(b) $\frac{\mu_0 i}{4\pi}.\frac{i}{r}(\pi + 2) \odot$

(c) $\frac{\mu_0 i}{4r} \otimes$

(d) $\frac{\mu_0 i}{4r} \odot$



(a) $\frac{\mu_0}{2r}\frac{2i}{r}(\pi + 1) \otimes$

(b) $\frac{\mu_0 i}{4r}.\frac{2i}{r}(\pi - 1) \otimes$

(c) Zero

(d) Infinite

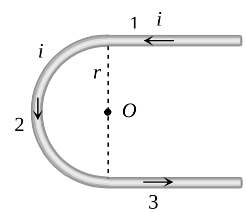
Solution : (i) (d) By using $B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \theta)i}{r} \Rightarrow B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)i}{r} = \frac{3\mu_0 i}{8r} \odot$

(ii) (b) Magnetic field at centre O due to section 1, 2 and 3 are respectively

$$B_1 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$$

$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r} \odot$$

$$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r} \odot$$

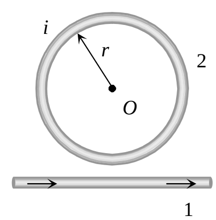


$$\Rightarrow B_{net} = B_1 + B_2 + B_3 = \frac{\mu_0}{4\pi}\frac{i}{r}(\pi + 2) \odot$$

(iii) (b) The given figure is equivalent to following figure, magnetic field at O due to long wire (part 1)

$$B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r} \odot$$

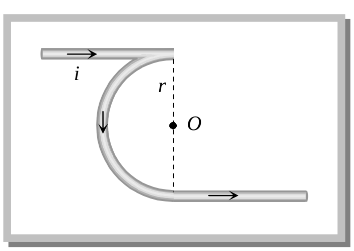
Due to circular coil $B_2 = \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} \otimes$



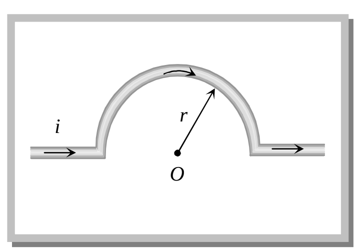
Hence net magnetic field at O

$$B_{net} = B_2 - B_1 = \frac{\mu_0}{4\pi}.\frac{2i}{r}(\pi - 1) \otimes$$

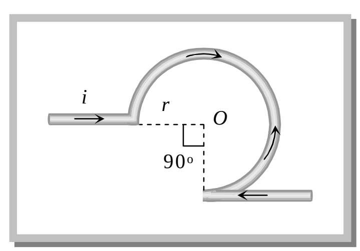
Example: 12 The field B at the centre of a circular coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents here shows three cases; in all cases the circular part has radius r and straight ones are infinitely long. For same current the field B is the centre P in cases 1, 2, 3 has the ratio[CPMT 198



(1)



(2)

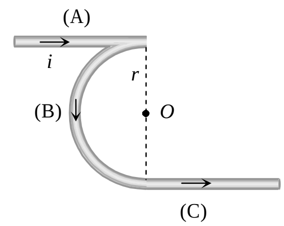


(3)

(a) $\left(-\frac{\pi}{2}\right):\left(\frac{\pi}{2}\right):\left(\frac{3\pi}{4}-\frac{1}{2}\right)$ (b) $\left(-\frac{\pi}{2}+1\right):\left(\frac{\pi}{2}+1\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

(c) $-\frac{\pi}{2}:\frac{\pi}{2}:\frac{3\pi}{4}$ (d) $\left(-\frac{\pi}{2}-1\right):\left(\frac{\pi}{2}-\frac{1}{2}\right):\left(\frac{3\pi}{4}+\frac{1}{2}\right)$

Solution : (a) *Case* 1 : $B_A = \frac{\mu_0}{4\pi}.\frac{i}{r}\otimes$



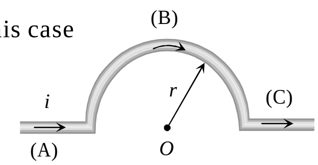
$$B_B = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 1

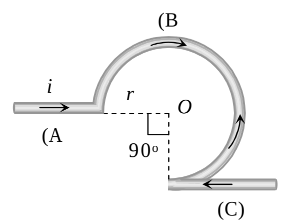
$$B_1 = B_B - (B_A + B_C) \quad \Rightarrow \quad B_1 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\odot \quad \text{..... (i)}$$

Case 2 : As we discussed before magnetic field at the centre O in this case



$$B_2 = \frac{\mu_0}{4\pi}.\frac{\pi i}{r}\otimes \quad \text{.....(ii)}$$

Case 3 : $B_A = 0$



$$B_B = \frac{\mu_0}{4\pi}.\frac{(2\pi - \pi/2)}{r}\otimes = \frac{\mu_0}{4\pi}.\frac{3\pi i}{2r}\otimes$$

$$B_C = \frac{\mu_0}{4\pi}.\frac{i}{r}\odot$$

So net magnetic field at the centre of case 3

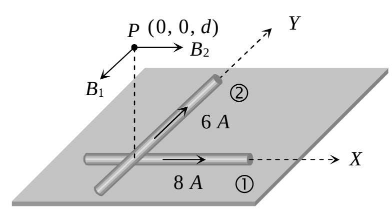
$$B_3 = \frac{\mu_0}{4\pi}.\frac{i}{r}\left(\frac{3\pi}{2}-1\right)\otimes \quad \text{....(iii)}$$

From equation (i), (ii) and (iii) $B_1 : B_2 : B_3 = \pi\odot : \pi\odot : \left(\frac{3\pi}{2}-1\right)\otimes = -\frac{\pi}{2}:\frac{\pi}{2}:\left(\frac{3\pi}{4}-\frac{1}{2}\right)$

Example: 13 Two infinite length wires carries currents $8A$ and $6A$ respectively and placed along X and Y-axis. Magnetic field at a point $P(0,0,d)m$ will be

(a) $\frac{7\mu_0}{\pi d}$ (b) $\frac{10\mu_0}{\pi d}$ (c) $\frac{14\mu_0}{\pi d}$ (d) $\frac{5\mu_0}{\pi d}$

Solution : (d) Magnetic field at P



Due to wire 1, $B_1 = \frac{\mu_0}{4\pi}.\frac{2(8)}{d}$

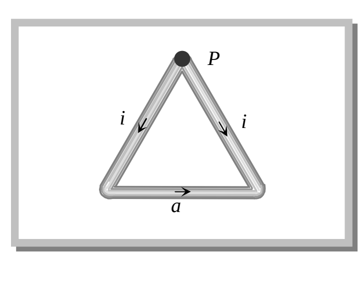
and due to wire 2, $B_2 = \frac{\mu_0}{4\pi}.\frac{2(16)}{d}$

$$\therefore \quad B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0}{4\pi}.\frac{16}{d}\right)^2 + \left(\frac{\mu_0}{4\pi}.\frac{12}{d}\right)^2} = \frac{\mu_0}{4\pi}\times\frac{2}{d}\times 10 = \frac{5\mu_0}{\pi d}$$

Example: 14 An equilateral triangle of side 'a' carries a current i then find out the magnetic field at point P which is vertex of triangle

(a) $\frac{\mu_0 i}{2\sqrt{3}\pi a}\otimes$

(b) $\frac{\mu_0 i}{2\sqrt{3}\pi a}\odot$

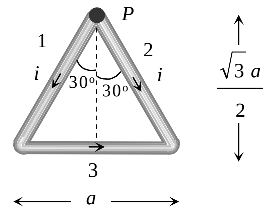


(c) $\frac{2\sqrt{3}\mu_0 i}{\pi a} \odot$

(d) Zero

Solution : (b) As shown in the following figure magnetic field at P due to side 1 and side 2 is zero.

Magnetic field at P is only due to side 3,



which is $B_1 = \frac{\mu_0}{4\pi}.\frac{2i\sin 30^o}{\frac{\sqrt{3}a}{2}} \odot$

$$= \frac{\mu_0}{4\pi}.\frac{2i}{\sqrt{3}a} \odot = \frac{\mu_0 i}{2\sqrt{3}\pi a} \odot$$

Example: 15 A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R. One of the arcs AB of the ring subtends an angle θ at the centre. The value of, the magnetic induction at the centre due to the current in the ring is [IIT-JEE 1995]

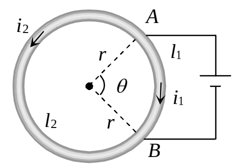
(a) Proportional to $2(180^o - \theta)$

(b) Inversely proportional to r

(c) Zero, only if $\theta = 180^o$

(d) Zero for all values of θ

Solution : (d) Directions of currents in two parts are different, so directions of magnetic fields due to these currents are different.



Also applying Ohm's law across AB $\quad i_1 R_1 = i_2 R_2 \Rightarrow i_1 l_1 = i_2 l_2$ .....(i)

Also $B_1 = \frac{\mu_0}{4\pi} \times \frac{i_1 l_1}{r^2}$ and $B_2 = \frac{\mu_0}{4\pi} \times \frac{i_2 l_2}{r^2}$; $\therefore \frac{B_2}{B_1} = \frac{i_1 l_1}{i_2 l_2} = 1$ [Using (i)]

Hence, two field are equal but of opposite direction. So, resultant magnetic induction at the centre is zero and is independent of θ.

Example: 16 The earth's magnetic induction at a certain point is $7 \times 10^{-5}\, Wb/m^2$. This is to be annulled by the magnetic induction at the centre of a circular conducting loop of radius 5 cm. The required current in the loop is [MP PET 1999; AIIMS 2000]

(a) 0.56 A (b) 5.6 A (c) 0.28 A (d) 2.8 A

Solution : (b) According to the question, at centre of coil $B = B_H \Rightarrow \frac{\mu_0}{4\pi}.\frac{2\pi i}{r} = B_H$

$$\Rightarrow 10^{-7} \times \frac{2\pi i}{(5 \times 1^{-2})} = 7 \times 10^{-5} \Rightarrow i = 5.6\ amp.$$

Example: 17 A particle carrying a charge equal to 100 times the charge on an electron is rotating per second in a circular path of radius 0.8 *metre*. The value of the magnetic field produced at the centre will be (μ_0 – permeability for vacuum) [CPMT 1986]

(a) $\frac{10^{-7}}{\mu_0}$ (b) $10^{-17}\mu_0$ (c) $10^{-6}\mu_0$ (d) $10^{-7}\mu_0$

Solution : (b) Magnetic field at the centre of orbit due to revolution of charge.

$B = \frac{\mu_0}{4\pi}.\frac{2\pi(q\nu)}{r}$; where ν = frequency of revolution of charge

So, $B = \frac{\mu_0}{4\pi} \times \frac{2\pi \times (100\, e \times 1)}{0.8} \Rightarrow B = 10^{-17}\mu_0$.

Example: 18 Ratio of magnetic field at the centre of a current carrying coil of radius R and at a distance of 3R on its axis is

(a) $10\sqrt{10}$ (b) $20\sqrt{10}$ (c) $2\sqrt{10}$ (d) $\sqrt{10}$

Solution : (a) By using $\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{r^2}\right)^{3/2}$; where x = 3R and r = R $\Rightarrow \frac{B_{centre}}{B_{axis}} = (10)^{3/2} = 10\sqrt{10}$.

Example: 19 A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be $\frac{1}{8}th$ to its value at the centre of the coil, is [MP PMT 1997]

(a) $\frac{R}{\sqrt{3}}$ (b) $R\sqrt{3}$ (c) $2\sqrt{3}R$ (d) $\frac{2}{\sqrt{3}}R$

Solution : (b) By using $\frac{B_{centre}}{B_{axis}} = \left(1+\frac{x^2}{r^2}\right)^{3/2}$, given $r = R$ and $B_{axis} = \frac{1}{8}B_{centre}$

$$\Rightarrow 8 = \left(1+\frac{x^2}{R^2}\right)^{3/2} \Rightarrow (2)^2 = \left\{\left(1+\frac{x^2}{R^2}\right)^{1/2}\right\}^3 \Rightarrow 2 = \left(1+\frac{x^2}{R^2}\right)^{1/2} \Rightarrow 4 = 1+\frac{x^2}{R^2} \Rightarrow x = \sqrt{3}R$$

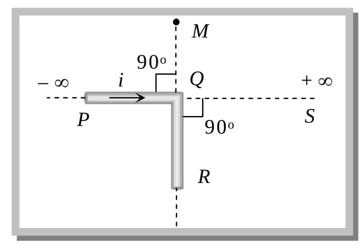
Example: 20 An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current is $\frac{1}{2}$ in QR as well as in QS, the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1 / H_2 is given by

(a) $\frac{1}{2}$

(b) 1

(c) $\frac{2}{3}$

(d) 2



Solution : (c) Magnetic field at any point lying on the current carrying conductor is zero.

Here H_1 = magnetic field at M due to current in PQ

H_2 = magnetic field at M due to R + due to QS + due to PQ $= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1$

$\therefore \quad \frac{H_1}{H_2} = \frac{2}{3}$

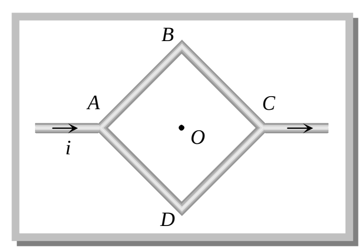
Example: 21 Figure shows a square loop $ABCD$ with edge length a. The resistance of the wire ABC is r and that of ADC is $2r$. The value of magnetic field at the centre of the loop assuming uniform wire is

(a) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \odot$

(b) $\frac{\sqrt{2}\,\mu_0 i}{3\pi a} \otimes$

(c) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \odot$

(d) $\frac{\sqrt{2}\,\mu_0 i}{\pi a} \otimes$

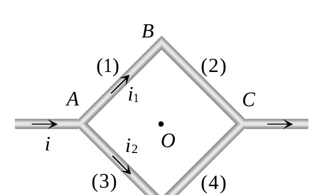


Solution : (b) According to question resistance of wire ADC is twice that of wire ABC. Hence current flows through ADC is half that of ABC *i.e.* $\frac{i_2}{i_1} = \frac{1}{2}$. Also $i_1 + i_2 = i \Rightarrow i_1 = \frac{2i}{3}$ and $i_2 = \frac{i}{3}$

Magnetic field at centre O due to wire AB and BC (part 1 and 2) $B_1 = \frac{\mu_0}{4\pi}.\frac{2i_1 \sin 45^o}{a/2} \otimes = \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\,i_1}{a} \otimes$

and magnetic field at centre O due to wires AD and DC (*i.e.* part 3 and 4) $B_3 = B_4 = \frac{\mu_0}{4\pi}\frac{2\sqrt{2}\,i_2}{a} \odot$

Also $i_1 = 2i_2$. So $(B_1 = B_2) > (B_3 = B_4)$



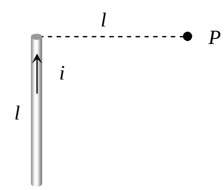
Hence net magnetic field at centre O

$$B_{net} = (B_1 + B_2) - (B_3 + B_4)$$

$$= 2 \times \frac{\mu_0}{4\pi}.\frac{2\sqrt{2} \times \left(\frac{2}{3}i\right)}{a} - \frac{\mu_0}{4\pi}.\frac{2\sqrt{2}\left(\frac{i}{3}\right) \times 2}{a} = \frac{\mu_0}{4\pi}.\frac{4\sqrt{2}\,i}{3a}(2-1)\otimes = \frac{\sqrt{2}\,\mu_0 i}{3\pi\, a}\otimes$$

Tricky example: 1

Figure shows a straight wire of length l current i. The magnitude of magnetic field produced by the current at point P is

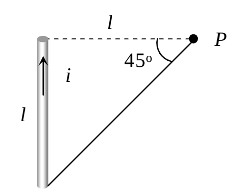


(a) $\frac{\sqrt{2}\mu_0 i}{\pi l}$ (b) $\frac{\mu_0 i}{4\pi l}$ (c) $\frac{\sqrt{2}\mu_0 i}{8\pi l}$ (d) $\frac{\mu_0 i}{2\sqrt{2}\pi l}$

Solution: (c) The given situation can be redrawn as follow.

As we know the general formula for finding the magnetic field due to a finite length wire

$$B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$$

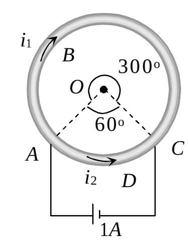


Here $\phi_1 = 0^o$, $\phi = 45^o$

$$\therefore\ B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin 0^o + \sin 45^o) = \frac{\mu_0}{4\pi}.\frac{i}{\sqrt{2}l} \Rightarrow B = \frac{\sqrt{2}\mu_0 i}{8\pi l}$$

Tricky example: 2

A cell is connected between the points A and C of a circular conductor $ABCD$ of centre 'O' with angle $AOC = 60°$, If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio $\frac{B_1}{B_2}$ is [KCET (Engg./ Med.) 1999]



(a) 0.2

(b) 6

(c) 1

(d) 5

Solution: (c) $B = \frac{\mu_0}{4\pi}.\frac{\theta\, i}{r}$

$\Rightarrow B \propto \theta\, i$

$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{i_1}{i_2}$

Also $\frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1}$ Hence $\frac{B_1}{B_2} = \frac{1}{1}$