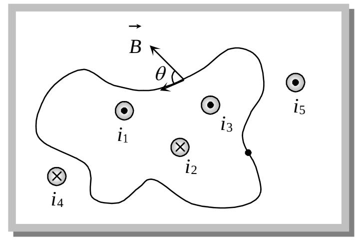
## Amperes Law

Amperes law gives another method to calculate the magnetic field due to a given current distribution.

Line integral of the magnetic field $\vec{B}$ around any closed curve is equal to $\mu_0$ times the net current $i$ threading through the area enclosed by the curve




*i.e.* $\oint \vec{B}d\vec{l} = \mu_0 \sum i = \mu_0 (i_1 + i_3 - i_2)$

Also using $\vec{B} = \mu_0 \vec{H}$ (where $\vec{H}$ = magnetising field)

$\oint \mu_0 \vec{H}.\vec{dl} = \mu_0 \Sigma i \Rightarrow \oint \vec{H}.\vec{dl} = \Sigma i$

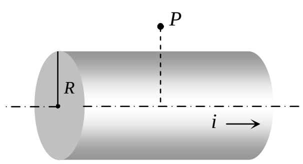
**Note :** ❒ Total current crossing the above area is $(i_1 + i_3 - i_2)$. Any current outside the area is not included in net current. (Outward $\odot \rightarrow +ve$, Inward $\otimes \rightarrow -ve$)

❒ When the direction of current is away from the observer then the direction of closed path is clockwise and when the direction of current is towards the observer then the direction of closed path is anticlockwise.

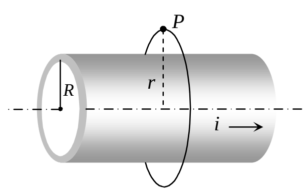

## Application of Amperes law

(1) Magnetic field due to a cylindrical wire

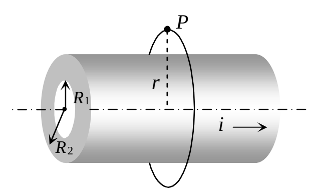
(i) Outside the cylinder



Solid cylinder



Thin hollow cylinder

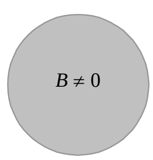


Thick hollow

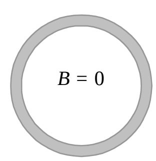
In all above cases magnetic field outside the wire at $P$ $\oint \vec{B}.\vec{dl} = \mu_0 i \Rightarrow B\int dl = \mu_0 i \Rightarrow B \times 2\pi r = \mu_0 i \Rightarrow$ $B_{out} = \dfrac{\mu_0 i}{2\pi r}$

In all the above cases $B_{surface} = \dfrac{\mu_0 i}{2\pi R}$

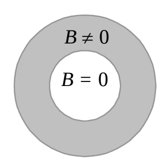
(ii) Inside the cylinder : Magnetic field inside the hollow cylinder is zero.



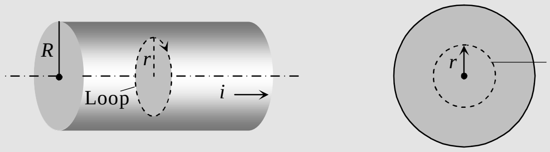
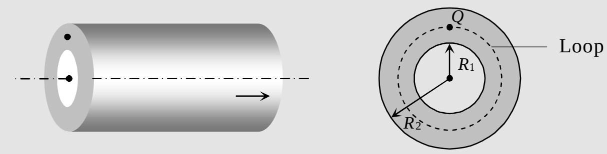
Cross sectional view Solid cylinder



Thin hollow cylinder



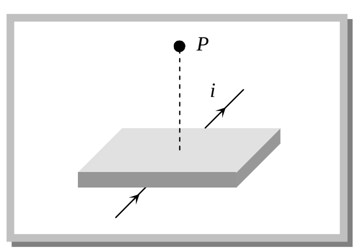
Thick hollow

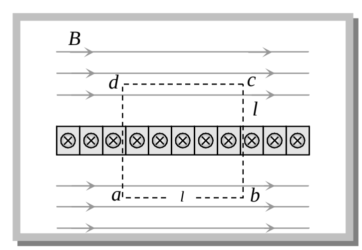
| Solid cylinder | Inside the thick portion of hollow cylinder |
|---|---|
|  |  |
| Current enclosed by loop ($i'$) is lesser then the total current ($i$) | Current enclosed by loop ($i'$) is lesser then the total current ($i$) |
| Current density is uniform *i.e.* $J = J' \Rightarrow \dfrac{i}{A} = \dfrac{i'}{A'}$ | Also $i' = i \times \dfrac{A'}{A} = i \times \dfrac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ |
| $\Rightarrow i' = i \times \dfrac{A'}{A} = i\left(\dfrac{r^2}{R^2}\right)$ | Hence at point $Q$ $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \mu_0 i \times \dfrac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$ |
| Hence at point $Q$ $\oint \vec{B}.d\vec{l} = \mu_0 i' \Rightarrow B \times 2\pi r = \dfrac{\mu_0 i r^2}{R^2}$ | $\Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.\dfrac{(r^2 - R_1^2)}{(R_2^2 - R_1^2)}$. If $r = R_1$ (inner surface) $B = 0$ |
| $\Rightarrow B = \dfrac{\mu_0}{2\pi}.\dfrac{ir}{R^2}$ | If $r = R_2$ (outer surface) $B = \dfrac{\mu_0 i}{2\pi R_2}$ (max.) |

**Note :** ❒ For all cylindrical current distributions

$B_{axis} = 0$ (min.), $B_{surface} =$ max (distance $r$ always from axis of cylinder), $B_{out} \propto 1/r$.

(2) Magnetic field due to an infinite sheet carrying current : The figure shows an infinite sheet of current with linear current density $j$ ($A/m$). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side $l$ as shown in the figure.




According to Ampere's law, $\int_a^b B.dl + \int_b^c B.dl + \int_c^d B.dl + \int_d^a B.dl = \mu_0 i$.

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c B.dl = 0$ ; $\int_d^a B.dl = 0$

Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b B.dl + \int_d^a B.dl = 2Bl$

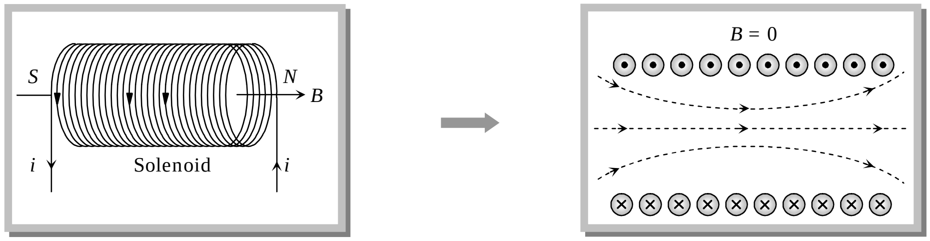
The current enclosed by the loop is $i = jl$

Therefore, according to Ampere's law $2Bl = \mu_0 (jl)$ or $B = \dfrac{\mu_0 j}{2}$

(3) Solenoid

A cylinderical coil of many tightly wound turns of insulated wire with generally diameter of the coil smaller than its length is called a solenoid.

One end of the solenoid behaves like the north pole and opposite end behaves like the south pole. As the length of the solenoid increases, the interior field becomes more uniform and the external field becomes weaker.

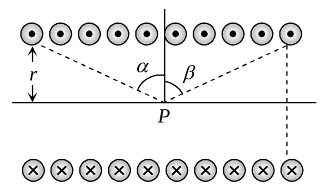



A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

(i) Finite length solenoid : If $N$ = total number of turns,

$l$ = length of the solenoid

$n$ = number of turns per unit length $= \frac{N}{l}$




Magnetic field inside the solenoid at point $P$ is given by $B = \frac{\mu_0}{4\pi}(2\pi ni)[\sin\alpha + \sin\beta]$

(ii) Infinite length solenoid : If the solenoid is of infinite length and the point is well inside the solenoid *i.e.* $\alpha = \beta = (\pi/2)$.

So $\boldsymbol{B_{in} = \mu_0 ni}$

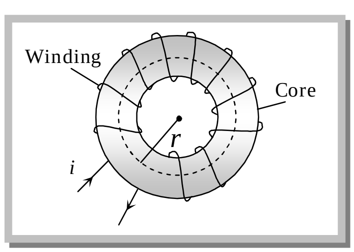
(ii) If the solenoid is of infinite length and the point is near one end *i.e.* $\alpha = 0$ and $\beta = (\pi/2)$

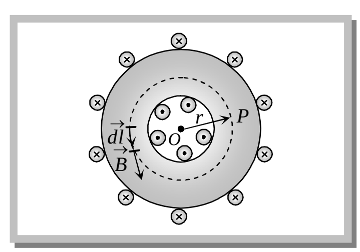
So $\boldsymbol{B_{end} = \frac{1}{2}(\mu_0 ni)}$

Note : ❐ Magnetic field outside the solenoid is zero.

❐ $B_{end} = \frac{1}{2}B_{in}$

(4) Toroid : A toroid can be considered as a ring shaped closed solenoid. Hence it is like an endless cylindrical solenoid.




Consider a toroid having $n$ turns per unit length

Let $i$ be the current flowing through the toroid (figure). The magnetic lines of force mainly remain in the core of toroid and are in the form of concentric circles. Consider such a circle of mean radius $r$. The circular closed path surrounds $N$ loops of wire, each of which carries a current $i$ therefore from $\oint \vec{B}.d\vec{l} = \mu_0 i_{net}$

$$\Rightarrow B \times (2\pi r) = \mu_0 Ni \qquad \Rightarrow B = \frac{\mu_0 Ni}{2\pi r} = \mu_o ni \text{ where } n = \frac{N}{2\pi r}$$

For any point inside the empty space surrounded by toroid and outside the toroid, magnetic field $B$ is zero because the net current enclosed in these spaces is zero.

## Concepts

- *The line integral of magnetising field $(\vec{H})$ for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.*
- *Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.*
- *Biot-Savart law is valid for asymmetrical current distributions while Ampere's law is valid for symmetrical current distributions.*
- *Biot-Savart law is based only on the principle of magnetism while Ampere's laws is based on the principle of electromagnetism.*

## Example

*Example*: 22 A long solenoid has 200 *turns per cm* and carries a current of 2.5 *A*. The magnetic field at its centre is $[\mu_0 = 4\pi \times 10^{-7}\ Wb/m^2]$ [MP PET 2000]

(a) $3.14 \times 10^{-2}\ Wb/m^2$ (b) $6.28 \times 10^{-2}\ Wb/m^2$ (c) $9.42 \times 10^{-2}\ Wb/m^2$ (d) $12.56 \times 10^{-2}\ Wb/m^2$

*Solution* : (b) $B = \mu_0 ni = 4\pi \times 10^{-7} \times \dfrac{200}{10^{-2}} \times 2.5 = 6.28 \times 10^{-2}\ Wb/m^2$.

*Example*: 23 A long solenoid is formed by winding 20 *turns/cm*. The current necessary to produce a magnetic field of 20 *millitesla* inside the solenoid will be approximately $\left(\dfrac{\mu_0}{4\pi} = 10^{-7}\ Tesla\text{ - }metre\ /\ ampere\right)$ [MP PMT 1994]

(a) 8.0 *A* (b) 4.0 *A* (c) 2.0 *A* (d) 1.0 *A*

*Solution* : (a) $B = \mu_0 ni$ ; where $n = \dfrac{20}{10}\dfrac{turn}{cm} = 2000\dfrac{turn}{m}$. So, $20 \times 10^{-5} = 4\pi \times 2000 \times i \Rightarrow i = 8A$.

*Example*: 24 Two solenoids having lengths $L$ and $2L$ and the number of loops $N$ and $4N$, both have the same current, then the ratio of the magnetic field will be [CPMT 1994]

(a) 1 : 2 (b) 2 : 1 (c) 1 : 4 (d) 4 : 1

*Solution* : (a) $B = \mu_0 \dfrac{N}{L} i \Rightarrow B \propto \dfrac{N}{L} \Rightarrow \dfrac{B_1}{B_2} = \dfrac{N_1}{N_2} \times \dfrac{L_2}{L_1} = \dfrac{N}{4N} \times \dfrac{2l}{L} = \dfrac{1}{2}$.

*Example*: 25 The average radius of a toroid made on a ring of non-magnetic material is 0.1 m and it has 500 turns. If it carries 0.5 ampere current, then the magnetic field produced along its circular axis inside the toroid will be

(a) $25 \times 10^{-2}\ Tesla$ (b) $5 \times 10^{-2}\ Tesla$ (c) $25 \times 10^{-4}\ Tesla$ (d) $5 \times 10^{-4}\ Tesla$

*Solution* : (d) $B = \mu_0 ni$ ; where $n = \dfrac{N}{2\pi R}$ $\therefore B = 4\pi \times 10^{-7} \times \dfrac{500}{2\pi \times 0.1} \times 0.5 = 5 \times 10^{-4}\ T$.

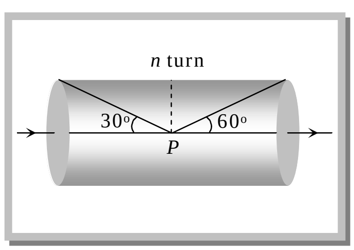
*Example*: 26 For the solenoid shown in figure. The magnetic field at point $P$ is

(a) $\dfrac{\mu_0 ni}{4}(\sqrt{3}+1)$

(b) $\dfrac{\sqrt{3}\mu_0 ni}{4}$

(c) $\dfrac{\mu_0 ni}{2}(\sqrt{3}+1)$

(d) $\dfrac{\mu_0 ni}{4}(\sqrt{3}-1)$



*Solution* : (a) $B = \frac{\mu_0}{4\pi}.2\pi ni(\sin\alpha + \sin\beta)$. From figure $\alpha = (90^o - 30^o) = 60^o$ and $\beta = (90^o - 60^o) = 30^o$

$\therefore\ B = \frac{\mu_0 ni}{2}(\sin 60^o + \sin 30^o) = \frac{\mu_0 ni}{4}(\sqrt{3}+1)$.

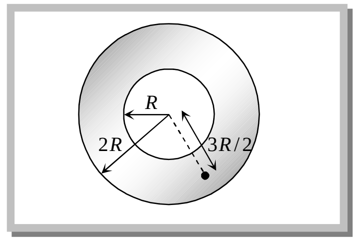
*Example*: 27 Figure shows the cress sectional view of the hollow cylindrical conductor with inner radius '$R$' and outer radius '$2R$', cylinder carrying uniformly distributed current along it's axis. The magnetic induction at point '$P$' at a distance $\frac{3R}{2}$ from the axis of the cylinder will be

(a) Zero

(b) $\frac{5\mu_0 i}{72\pi R}$

(c) $\frac{7\mu_0 i}{18\pi R}$

(d) $\frac{5\mu_0 i}{36\pi R}$




*Solution* : (d) By using $B = \frac{\mu_0 i}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$ here $r = \frac{3R}{2}$, $a = R$, $ab = 2R \Rightarrow$ $B = \frac{\mu_0 i}{2\pi\left(\frac{3R}{2}\right)} \times \left\{\frac{\left(\frac{3R}{2}\right) - R^2}{(R^2) - R^2}\right\} = \frac{5.\mu_o i}{36\pi r}$.

**Tricky example: 3**

A winding wire which is used to frame a solenoid can bear a maximum 10 $A$ current. If length of solenoid is 80$cm$ and it's cross sectional radius is 3 $cm$ then required length of winding wire is ($B = 0.2\,T$)

(a) $1.2 \times 10^2\ m$ (b) $4.8 \times 10^2\ m$ (c) $2.4 \times 10^3\ m$ (d) $6 \times 10^3\ m$

*Solution* : (c) $B = \frac{\mu_0 Ni}{l}$ where $N$ = Total number of turns, $l$ = length of the solenoid

$\Rightarrow\ 0.2 = \frac{4\pi \times 10^{-7} \times N \times 10}{0.8} \Rightarrow N = \frac{4 \times 10^4}{\pi}$

Since $N$ turns are made from the winding wire so length of the wire $(L) = 2\pi r \times N$ $[2\pi r = \text{length of each turns}]$

$\Rightarrow L = 2\pi \times 3 \times 10^{-2} \times \frac{4 \times 10^4}{\pi} = 2.4 \times 10^3\ m.$