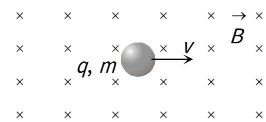
Motion of Charged Particle in a Magnetic Field

If a particle carrying a positive charge q and moving with velocity v enters a magnetic field B then it experiences a force F which is given by the expression

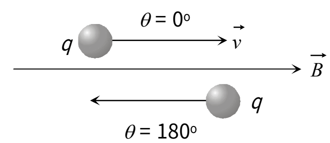


$$F = q(\vec{v} \times \vec{B}) \Rightarrow F = qvB\sin\theta$$

Here $\vec{v} =$ velocity of the particle, $\vec{B} =$ magnetic field

(1) **Zero force**

Force on charged particle will be zero (*i.e.* $F = 0$) if



(i) No field *i.e.* $B = 0 \Rightarrow F = 0$

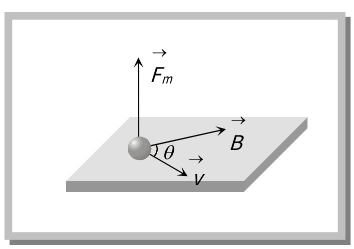
(ii) Neutral particle *i.e.* $q = 0 \Rightarrow F = 0$

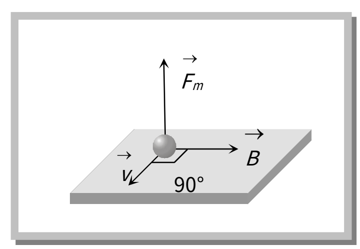
(iii) Rest charge *i.e.* $v = 0 \Rightarrow F = 0$

(iv) Moving charge *i.e.* $\theta = 0^o$ or $\theta = 180^o \Rightarrow F = 0$

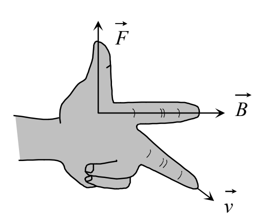
(2) **Direction of force**

The force $\vec{F}$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$ in accordance with Right Hand Screw Rule, through $\vec{v}$ and $\vec{B}$ themselves may or may not be perpendicular to each other.





Direction of force on charged particle in magnetic field can also be find by Flemings Left Hand Rule (FLHR).



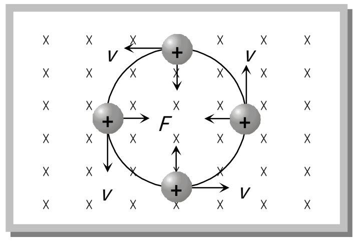
Here, *First finger* (indicates) $\rightarrow$ Direction of magnetic field

Middle finger $\rightarrow$ Direction of motion of positive charge or direction, opposite to the motion of negative charge.

Thumb $\rightarrow$ Direction of force

(3) **Circular motion of charge in magnetic field**

Consider a charged particle of charge q and mass m enters in a uniform magnetic field B with an initial velocity v perpendicular to the field.



θ = 90°, hence from $F = qvB \sin\theta$ particle will experience a maximum magnetic force $F_{max} = qvB$ which act's in a direction perpendicular to the motion of charged particle. (By Flemings left hand rule).

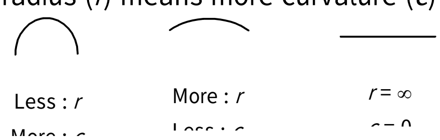
(i) **Radius of the path :** In this case path of charged particle is circular and magnetic force provides the necessary centripetal force *i.e.* $qvB = \frac{mv^2}{r} \Rightarrow$ radius of path $r = \frac{mv}{qB}$

If p = momentum of charged particle and K = kinetic energy of charged particle (gained by charged particle after accelerating through potential difference V) then $p = mv = \sqrt{2mK} = \sqrt{2mqV}$

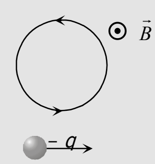
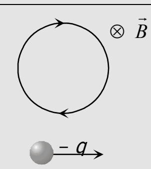
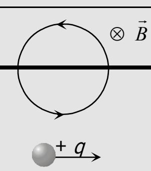
So $$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

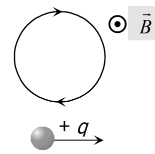
$r \propto v \propto p \propto \sqrt{K}$ *i.e.* with increase in speed or kinetic energy, the radius of the orbit increases.

Note : ❒ Less radius (r) means more curvature (c) *i.e.* $c \propto \frac{1}{r}$



(ii) **Direction of path :** If a charge particle enters perpendicularly in a magnetic field, then direction of path described by it will be

Type of charge	Direction of magnetic field	Direction of it's circular motion
Negative	Outwards ⊙	 Anticlockwise
Negative	Inward ⊗	 Clockwise
Positive	Inward ⊗	

		Anticlockwise
Positive	Outward ⊙	 Clockwise

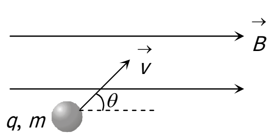
(iii) **Time period** : As in uniform circular motion $v = r\omega$, so the angular frequency of circular motion, called cyclotron or gyro-frequency, will be given by $\omega = \frac{v}{r} = \frac{qB}{m}$ and hence the time period, $T = \frac{2\pi}{\omega} = 2\pi \frac{m}{qB}$

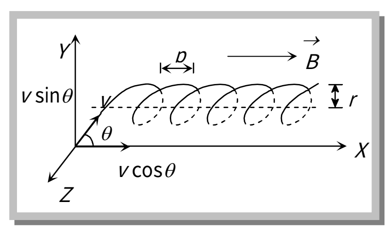
i.e., time period (or frequency) is independent of speed of particle and radius of the orbit and depends only on the field B and the nature, *i.e.*, specific charge $\left(\frac{q}{m}\right)$, of the particle.

(4) **Motion of charge on helical path**

When the charged particle is moving at an angle to the field (other than 0°, 90°, or 180°).

In this situation resolving the velocity of the particle along and perpendicular to the field, we find that the particle moves with constant velocity $v\cos\theta$ along the field (as no force acts on a charged particle when it moves parallel to the field) and at the same time it is also moving with velocity $v\sin\theta$ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field) of radius. $r = \frac{m(v\sin\theta)}{qB}$





Time period and frequency do not depend on velocity and so they are given by $T = \frac{2\pi m}{qB}$ and $\nu = \frac{qB}{2\pi m}$

So the resultant path will be a *helix* with its axis parallel to the field $\vec{B}$ as shown in figure in this situation.

The *pitch* of the *helix*, (*i.e.*, linear distance travelled in one rotation) will be given by $p = T(v\cos\theta) = 2\pi \frac{m}{qB}(v\cos\theta)$

Note : ❒ 1 rotation ≡ 2π ≡ T and 1 pitch ≡ 1 T

❒ Number of pitches ≡ Number of rotations ≡ Number of repetition = Number of helical turns

❒ If pitch value is p, then number of pitches obtained in length l given as

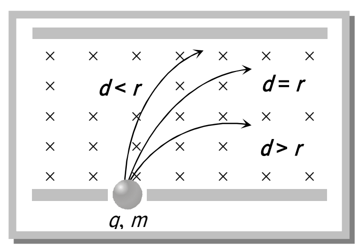
Number of pitches $= \frac{l}{p}$ and time reqd. $t = \frac{l}{v\cos\theta}$

Some standard results

& Ratio of radii of path described by proton and α-particle in a magnetic field (particle enters perpendicular to the field)

Constant quantity	Formula	Ratio of radii	Ratio of curvature (c)
v - same	$r = \frac{mv}{qB} \Rightarrow r \propto \frac{m}{q}$	$r_p : r_\alpha = 1 : 2$	$c_p : c_R = 2 : 1$
p - same	$r = \frac{p}{qB} \Rightarrow r \propto \frac{1}{q}$	$r_p : r_\alpha = 2 : 1$	$c_p : c_R = 1 : 2$
k - same	$r = \frac{\sqrt{2mk}}{qB} \Rightarrow r \propto \frac{\sqrt{m}}{q}$	$r_p : r_\alpha = 1 : 1$	$c_p : c_R = 1 : 1$
V - same	$r \propto \sqrt{\frac{m}{q}}$	$r_p : r_\alpha = 1 : \sqrt{2}$	$c_p : c_R = \sqrt{2} : 1$

& **Particle motion between two parallel plates** $(\vec{v} \perp \vec{B})$



(i) To strike the opposite plate it is essential that $d < r$.

(ii) Does not strike the opposite plate $d > r$.

(iii) To touch the opposite plate $d = r$.

(iv) To just not strike the opposite plate $d \geq r$.

(v) To just strike the opposite plate $d \leq r$.

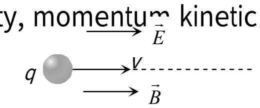
(5) **Lorentz force**

When the moving charged particle is subjected simultaneously to both electric field $\vec{E}$ and magnetic field $\vec{B}$, the moving charged particle will experience electric force $\vec{F}_e = q\vec{E}$ and magnetic force $\vec{F}_m = q(\vec{v} \times \vec{B})$; so the net force on it will be $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$. Which is the famous 'Lorentz-force equation'.

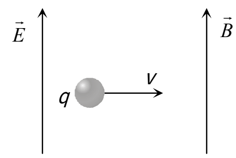
Depending on the directions of $\vec{v}, E$ and $\vec{B}$ following situations are possible

(i) **When** $\vec{v}, \vec{E}$ **and** $\vec{B}$ **all the three are collinear :** In this situation as the particle is moving parallel or antiparallel to the field, the magnetic force on it will be zero and only electric force will act and so $\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$

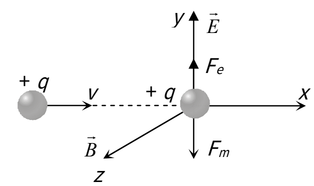
The particle will pass through the field following a straight line path (parallel field) with change in its speed. So in this situation speed, velocity, momentum kinetic energy all will change without change in direction of motion as shown



(ii) **When $\vec{E}$ is parallel to $\vec{B}$ and both these fields are perpendicular to $\vec{v}$ then :** $\vec{F_e}$ is perpendicular to $\vec{F_m}$ and they cannot cancel each other. The path of charged particle is curved in both these fields.



(iii) **$\vec{v}$, $\vec{E}$ and $\vec{B}$ are mutually perpendicular :** In this situation if $\vec{E}$ and $\vec{B}$ are such that



$$\vec{F} = \vec{F_e} + \vec{F_m} = 0 \text{ i.e., } \vec{a} = (\vec{F}/m) = 0$$

as shown in figure, the particle will pass through the field with same velocity.

And in this situation, as $F_e = F_m$ i.e., $qE = qvB$ $v = E/B$

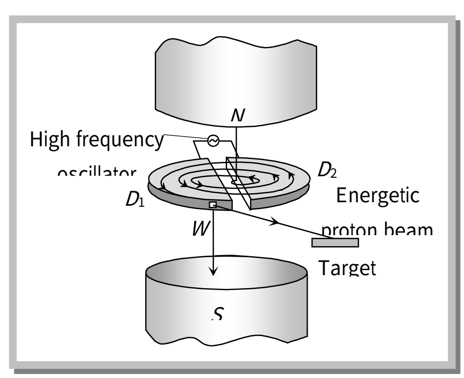
This principle is used in 'velocity-selector' to get a charged beam having a specific velocity.

Note : ❒ From the above discussion, conclusion is as follows

- ❒ If $E = 0$, $B = 0$, so $F = 0$.
- ❒ If $E = 0$, $B \neq 0$, so F may be zero (if $\theta = 0^o$ or 180^o).
- ❒ If $E \neq 0$, $B \neq 0$, so $F = 0$ (if $|\vec{F_e}| = |\vec{F_m}|$ and their directions are opposite)
- ❒ If $E \neq 0$, $B = 0$, so $F \neq 0$ (because $\vec{v} \neq \text{constant}$).

Cyclotron

Cyclotron is a device used to accelerated positively charged particles (like, α-particles, deutrons *etc.*) to acquire enough energy to carry out nuclear disintegration *etc.* t is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency. Thus a small potential difference would impart if enormously large velocities if the particle is made to traverse the potential difference a number of times.



It consists of two hollow *D*-shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} *mm* mercury. The whole apparatus is placed between the two poles of a strong electromagnet *NS* as shown in fig. The magnetic field acts perpendicular to the plane of the dees.

Note : ❒ The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.

(1) **Cyclotron frequency :** Time taken by ion to describe *q* semicircular path is given by $t = \frac{\pi r}{v} = \frac{\pi m}{qB}$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of position :** Maximum energy gained by the charged particle $E_{\max} = \left(\frac{q^2B^2}{2m}\right)r^2$

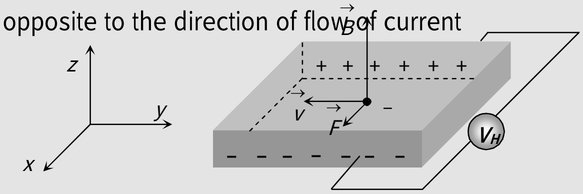
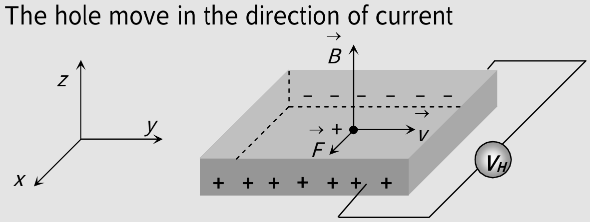
where r_0 = maximum radius of the circular path followed by the positive ion.

Note : ❒ Cyclotron frequency is also known as magnetic resonance frequency.

❒ Cyclotron can not accelerate electrons because they have very small mass.

Hall effect : The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

Hall effect helps us to know the nature and number of charge carriers in a conductor.

Negatively charged particles	**Positively charged particles**
Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current 	Let the current carriers be positively charged holes. The hole move in the direction of current 
force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along *x*-axis and hence electrons will move towards face (2) and it becomes negatively charged.	Force acting on the hole due to magnetic field $\vec{F_m} = +e(\vec{v} \times \vec{B})$ force acts along *x*-axis and hence holes move towards face (2) and it becomes positively charged.

Concepts

- *The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.*
- *Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.*
- *Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.*
- *If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit)*
- ***Deviation of charged particle in magnetic field**: If a charged particle (q, m) enters a uniform magnetic field $\vec{B}$ (extends upto a length x) at right angles with speed v as shown in figure.*

  The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.

  Deviation in terms of time t; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

  Deviation in terms of length of the magnetic field ; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. *This relation can be used only when* $x \le r$.

  For x > r, the deviation will be 180° as shown in the following figure

Examples

***Example*: 28** Electrons move at right angles to a magnetic field of 1.5×10^{-2} *Tesla* with a speed of 6×10^{27} m/s. If the specific charge of the electron is 1.7×10^{11} *Coul/kg*. The radius of the circular path will be **[BHU 2003]**

(a) 2.9 *cm* (b) 3.9 *cm* (c) 2.35 *cm* (d) 3 *cm*

Solution: (c) $r = \frac{mv}{qB} \Rightarrow \frac{v}{(q/m).B} = \frac{6 \times 10^{27}}{17 \times 10^{11} \times 1.5 \times 10^{-2}} = 2.35 \times 10^{-2}\ m = 2.35\ cm.$

***Example*: 29** An electron (mass $= 9 \times 10^{-31}\ kg.$ charge $= 1.6 \times 10^{-19}\ coul.$) whose kinetic energy is 7.2×10^{-18} *joule* is moving in a circular orbit in a magnetic field of $9 \times 10^{-5}\ weber/m^2$. The radius of the orbit is **[MP PMT 2002]**

(a) 1.25 *cm* (b) 2.5 *cm* (c) 12.5 *cm* (d) 25.0 *cm*

Solution: (d) $r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2 \times q \times 10^{-31} \times 7.2 \times 10^{-8}}}{1.6 \times 10^{-19} \times q \times 10^{-5}} = 0.25\ cm = 25\ cm$.

Example: 30 An electron and a proton enter a magnetic field perpendicularly. Both have same kinetic energy. Which of the following is true **[MP PET 1999]**

(a) Trajectory of electron is less curved (b) Trajectory of proton is less curved

(c) Both trajectories are equally curved (d) Both move on straight line path

Solution : (b) By using $r = \frac{\sqrt{2mk}}{qB}$; For both particles $q \to$ same, $B \to$ same, $k \to$ same

Hence $r \propto \sqrt{m} \Rightarrow \frac{r_e}{r_p} = \sqrt{\frac{m_e}{m_p}}$ $\because m_p > m_e$ so $r_p > r_e$

Since radius of the path of proton is more, hence it's trajectory is less curved.

Example: 31 A proton and an $\alpha -$particles enters in a uniform magnetic field with same velocity, then ratio of the radii of path describe by them

(a) $1:1$ (b) $1:2$ (c) $2:1$ (d) None of these

Solution : (b) By using $r = \frac{mv}{qB}$; $v \to$ same, $B \to$ same $\Rightarrow r \propto \frac{m}{2} \Rightarrow \frac{r_p}{r_\alpha} = \frac{m_p}{m_\alpha} \times \frac{q_\alpha}{q_p} = \frac{m_p}{4m_p} \times \frac{2q_p}{q_p} = \frac{1}{2}$

Example: 32 A proton of mass *m* and charge $+e$ is moving in a circular orbit of a magnetic field with energy 1*MeV.* What should be the energy of α-particle (mass = 4 *m* and charge = +2*e*), so that it can revolve in the path of same radius **[BHU 1997]**

(a) 1 *MeV* (b) 4 *MeV* (c) 2 *MeV* (d) 0.5 *MeV*

Solution : (a) By using $r = \frac{\sqrt{2mK}}{qB}$; $r \to$ same, $B \to$ same $\Rightarrow K \propto \frac{q^2}{m}$

Hence $\frac{K_\alpha}{K_p} = \left(\frac{q_\alpha}{q_p}\right)^2 \times \frac{m_p}{m_\alpha} = \left(\frac{2q_p}{q_p}\right)^2 \times \frac{m_p}{4m_p} 1 \Rightarrow K_\alpha = K_p = 1\, meV$.

Example: 33 A proton and an $\alpha -$particle enter a uniform magnetic field perpendicularly with the same speed. If proton takes $25\,\mu\,\text{sec}$ to make 5 revolutions, then the periodic time for the $\alpha -$particle would be **[MP PET 1993]**

(a) $50\,\mu$ sec (b) $25\,\mu$ sec (c) $10\,\mu$ sec (d) $5\,\mu$ sec

Solution : (c) Time period of proton $T_p = \frac{25}{5} = 5\,\mu\,\text{sec}$

By using $T = \frac{2\pi m}{qB} \Rightarrow \frac{T_\alpha}{T_p} = \frac{m_\alpha}{m_p} \times \frac{q_p}{q_\alpha} = \frac{4m_p}{m_p} \times \frac{q_p}{2q_p} \Rightarrow T_\alpha = 2T_p = 10\,\mu$ sec .

Example: 34 A particle with 10^{-11} *coulomb* of charge and 10^{-7} *kg* mass is moving with a velocity of 10^8 *m/s* along the *y*-axis. A uniform static magnetic field *B* = 0.5 *Tesla* is acting along the *x*-direction. The force on the particle is

[MP PMT 1997]

(a) 5×10^{-11} *N* along $\hat{i}$ (b) 5×10^{3} *N* along $\hat{k}$ (c) 5×10^{-11} *N* along $-\hat{j}$ (d) 5×10^{-4} *N* along $-\hat{k}$

Solution : (d) By using $\vec{F} = q(\vec{v} \times \vec{B})$; where $\vec{v} = 10\hat{j}$ and $\vec{B} = 0.5\hat{i}$

$\Rightarrow \vec{F} = 10^{-11}(10^{8}\hat{j} \times 0.5\hat{i}) = 5 \times 10^{-4}(\hat{j} \times \hat{i}) = 5 \times 10^{-4}(-\hat{k})$ *i.e.,* 5×10^{-4} *N* along $-\hat{k}$.

***Example*: 35** An electron is moving along positive *x*-axis. To get it moving on an anticlockwise circular path in *x-y* plane, a magnetic filed is applied

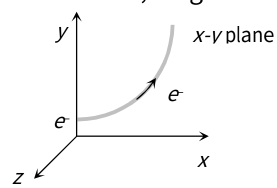
(a) Along positive *y*-axis (b) Along positive *z*-axis

(c) Along negative *y*-axis (d) Along negative *z*-axis

Solution : (a) The given situation can be drawn as follows

According to figure, for deflecting electron in *x-y* plane, force must be acting an it towards *y*-axis.

Hence according to Flemings left hand rule, magnetic field directed along positive y – axis.



***Example*: 36** A particle of charge -16×10^{-18} coulomb moving with velocity 10 *m/s* along the *x*-axis enters a region where a magnetic field of induction *B* is along the *y*-axis, and an electric field of magnitude 10^4 *V/m* is along the negative *z*-axis. If the charged particle continuous moving along the *x*-axis, the magnitude of *B* is **[AIEEE 2003]**

(a) $10^{-3}\ Wb/m^2$ (b) $10^{3}\ Wb/m^2$ (c) $10^{5}\ Wb/m^2$ (d) $10^{16}\ Wb/m^2$

Solution : (b) Particles is moving undeflected in the presence of both electric field as well as magnetic field so it's speed

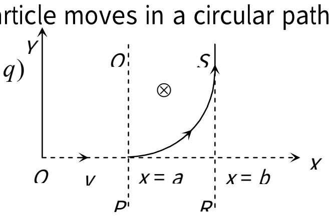
$v = \frac{E}{B} \Rightarrow B = \frac{E}{v} = \frac{10^4}{10} = 10^3\ Wb/m^2$.

***Example*: 37** A particle of mass *m* and charge *q* moves with a constant velocity *v* along the positive *x* direction. It enters a region containing a uniform magnetic field *B* directed along the negative *z* direction extending from *x* = *a* to *x* = *b*. The minimum value of *v* required so that the particle can just enter the region *x* > *b* is

[IIT-JEE (Screening) 2002]

(a) *qbB/m* (b) *q*(*b* – *a*)*B/m* (c) *qaB/m* (d) *q(b+a)B/2m*

Solution : (b) As shown in the following figure, the $z-$axis points out of the paper and the magnetic fields is directed into the paper, existing in the region between *PQ* and *RS*. The particle moves in a circular path of radius *r* in the magnetic field. It can just enter the region $x > b$ for $r \geq (b-q)$



Now $r = \dfrac{mv}{qb} \geq (b-a)$

$\Rightarrow v \geq \dfrac{q(b-a)B}{m} \quad \Rightarrow v_{\min} = \dfrac{q(b-a)B}{m}$.

***Example*: 38** At a certain place magnetic field vertically downwards. An electron approaches horizontally towards you and enters in this magnetic fields. It's trajectory, when seen from above will be a circle which is

(a) Vertical clockwise (b) Vertical anticlockwise

(c) Horizontal clockwise (d) Horizontal anticlockwise

Solution : (c) By using Flemings left hand rule.

***Example*: 39** When a charged particle circulates in a normal magnetic field, then the area of it's circulation is proportional to

(a) It's kinetic energy (b) It's momentum

(c) It's charge (d) Magnetic fields intensity

Solution : (a) $r = \dfrac{\sqrt{2mK}}{qB}$ and $A = Aq^2 \Rightarrow A = \dfrac{\pi(2mK)}{q^2b^2} \Rightarrow A \propto K.$

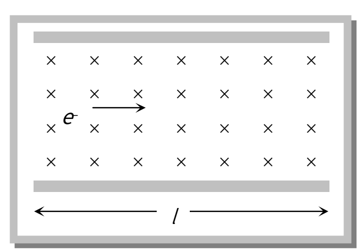
***Example*: 40** An electron moves straight inside a charged parallel plate capacitor at uniform charge density σ. The space between the plates is filled with constant magnetic field of induction $\vec{B}$. Time of straight line motion of the electron in the capacitor is

(a) $\dfrac{e\sigma}{\varepsilon_0 lB}$

(b) $\dfrac{\varepsilon_0 lB}{\sigma}$

(c) $\dfrac{e\sigma}{\varepsilon_0 B}$

(d) $\dfrac{\varepsilon_0 B}{e\sigma}$



Solution : (b) The net force acting on the electron is zero because it moves with constant velocity, due to it's motion on straight line.

$\Rightarrow \vec{F}_{net} = \vec{F}_e + \vec{F}_m = 0 \Rightarrow |\vec{F}_e| = |\vec{F}_m| \Rightarrow eE = evB \Rightarrow ve = \dfrac{E}{B} = \dfrac{\sigma}{\varepsilon_0 B} \qquad \left[E = \dfrac{\sigma}{\varepsilon_o}\right]$

$\therefore$ The time of motion inside the capacitor $t = \dfrac{l}{v} = \dfrac{\varepsilon_0 lB}{\sigma}$.

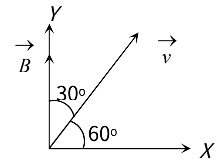
Example: 41 A proton of mass 1.67×10^{-27} *kg* and charge 1.6×10^{-19} *C* is projected with a speed of 2×10^6 m/s at an angle of 60^0 to the *X*-axis. If a uniform magnetic field of 0.104 *Tesla* is applied along *Y*-axis, the path of proton is

(a) A circle of radius = 0.2 *m* and time period $\pi \times 10^{-7}$ *s*

(b) A circle of radius = 0.1 *m* and time period $2\pi \times 10^{-7}$ *s*

(c) A helix of radius = 0.1 *m* and time period $2\pi \times 10^{-7}$ *s*

(d) A helix of radius = 0.2 *m* and time period $4\pi \times 10^{-7}$ *s*



Solution : (b) By using $r = \frac{mv \sin\theta}{qB} \Rightarrow r = \frac{1.67 \times 15^{27} \times 2 \times 10^6 \times \sin 30°}{1.6 \times 10^{-19} \times 0.104} = 0.1m$

and it's time period $T = \frac{2\pi m}{qB} = \frac{2 \times \pi \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19} \times 0.104} = 2\pi \times 10^{-7}$ sec .

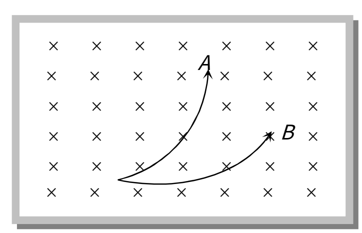
Example: 42 A charge particle, having charge *q* accelerated through a potential difference *V* enter a perpendicular magnetic field in which it experiences a force *F*. If *V* is increased to 5 *V*, the particle will experience a force

(a) *F* (b) 5*F* (c) $\frac{F}{5}$ (d) $\sqrt{5}F$

Solution : (d) $\frac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\frac{2qV}{m}}$. Also $F = qvB$

$\Rightarrow F = qB\sqrt{\frac{2qV}{m}}$ hence $F \propto \sqrt{V}$ which gives $F' = \sqrt{5}F$.

Example: 43 The magnetic field is downward perpendicular to the plane of the paper and a few charged particles are projected in it. Which of the following is true **[CPMT 1997]**



(a) A represents proton and *B* and electron

(b) Both *A* and *B* represent protons but velocity of *A* is more than that of *B*

(c) Both *A* and *B* represents protons but velocity of *B* is more than that of *A*

(d) Both *A* and *B* represent electrons, but velocity of *B* is more than that of *A*

Solution : (c) Both particles are deflecting in same direction so they must be of same sign.(*i.e.,* both *A* and *B* represents protons)

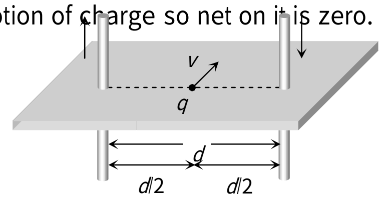
By using $r = \frac{mv}{qB} \Rightarrow r \propto v$

From given figure radius of the path described by particle *B* is more than that of *A*. Hence $v_B > v_A$.

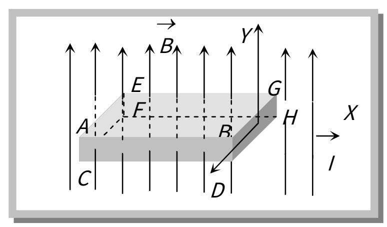
Example: 44 Two very long straight, particle wires carry steady currents *i* and – *i* respectively. The distance between the wires is *d.* At a certain instant of time, a point charge *q* is at a point equidistant from the two wires, in the plane of the wires. It's instantaneous velocity $\vec{v}$ is perpendicular to this plane. The magnitude of the force due to the magnetic field acting on the charge at this instant is

(a) $\frac{\mu_0 iqv}{2\pi d}$ (b) $\frac{\mu_0 iqv}{\pi d}$ (c) $\frac{2\mu_0 iqv}{\pi d}$ (d) Zero

Solution : (d) According to gives information following figure can be drawn, which shows that direction of magnetic field is along the direction of motion of charge so net on it is zero.



Example: 45 A metallic block carrying current *i* is subjected to a uniform magnetic induction $\vec{B}$ as shown in the figure. The moving charges experience a force *F* given by ……. which results in the lowering of the potential of the face ……. Assume the speed of the carriers to be v



(a) $eVB\hat{k}$, $ABCD$

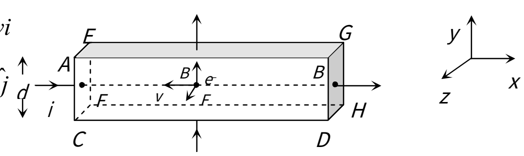
(b) $eVB\hat{k}$, $ABCD$

(c) $-eVB\hat{k}$, $ABCD$

(d) $-eVB\hat{k}$, $EFGH$

Solution : (c) As the block is of metal, the charge carriers are electrons; so for current along positive *x*-axis, the electrons are moving along negative *x*-axis, *i.e.* $\vec{v} = -vi$

and as the magnetic field is along the *y*-axis, *i.e.* $\vec{B} = B\hat{j}$



so $\vec{F} = q(\vec{v} \times \vec{B})$ for this case yield $\vec{F} = (-e)[-v\hat{i} \times B\hat{j}]$

i.e., $\vec{F} = evB\,\hat{k}$ [As $\hat{i} \times \hat{j} = \hat{k}$]

As force on electrons is towards the face *ABCD*, the electrons will accumulate on it an hence it will acquire lower potential.

Tricky example: 4

An ionised gas contains both positive and negative ions. If it is subjected simultaneously to an electric field along the +*ve x*-axis and a magnetic field along the +*z* direction then **[IIT-JEE (Screening) 20**

(a) Positive ions deflect towards +*y* direction and negative ions towards – *y* direction

(b) All ions deflect towards + *y* direction

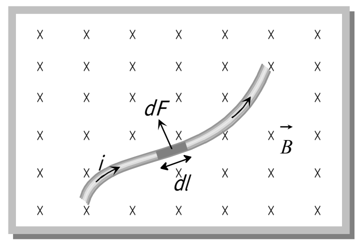
(c) All ions deflect towards – *y* direction

(d) Positive ions deflect towards – *y* direction and negative ions towards + *y* direction.

Solution : (c) As the electric field is switched on, positive ion will start to move along positive *x*-direction and negative ion along negative *x*-direction. Current associated with motion of both types of ions is along positive *x*-direction. According to Flemings left hand rule force on both types of ions will be along negative *y*-direction.

Force on a Current Carrying Conductor in Magnetic Field

In case of current carrying conductor in a magnetic field force experienced by its small length element is $d\vec{F} = i d\vec{l} \times \vec{B}$; $i d\vec{l}$ = current element $d\vec{F} = I(d\vec{l} \times \vec{B})$



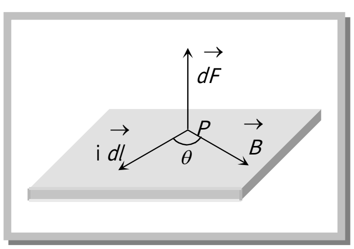
Total magnetic force $\vec{F} = \int d\vec{F} = \int i(d\vec{l} \times \vec{B})$

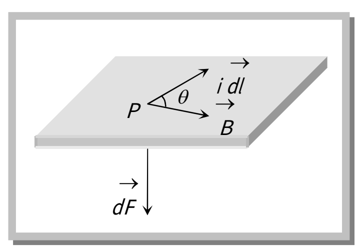
If magnetic field is uniform *i.e.*, $\vec{B}$ = constant

$$\vec{F} = i\left[\int d\vec{l}\right] \times \vec{B} = i(\vec{L}' \times \vec{B})$$

$\int d\vec{l} = \vec{L} =$ vector sum of all the length elements from initial to final point. Which is in accordance with the law of vector addition is equal to length vector $\vec{L}'$ joining initial to final point.

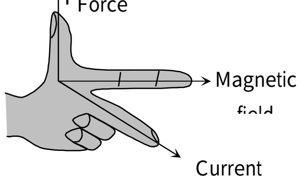
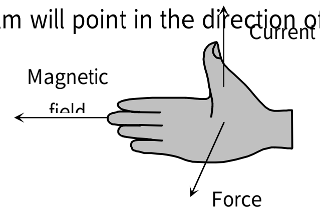
(1) **Direction of force :** The direction of force is always perpendicular to the plane containing $i d\vec{l}$ and $\vec{B}$ and is same as that of cross-product of two vectors $(\vec{A} \times \vec{B})$ with $\vec{A} = i d\vec{l}$.



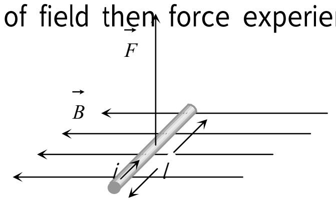


The direction of force when current element $i d\vec{l}$ and $\vec{B}$ are perpendicular to each other can also be determined by applying either of the following rules

Fleming's left-hand rule	Right-hand palm rule

Stretch the fore-finger, central finger and thumb left hand mutually perpendicular. Then if the fore-finger points in the direction of field $\vec{B}$ and the central in the direction of current *i*, the thumb will point in the direction of force 	Stretch the fingers and thumb of right hand at right angles to each other. Then if the fingers point in the direction of field $\vec{B}$ and thumb in the direction of current *i*, then normal to the palm will point in the direction of force 

(2) **Force on a straight wire :** If a current carrying straight conductor (length *l*) is placed in an uniform magnetic field (*B*) such that it makes an angle θ with the direction of field then force experienced by it is $F = Bil\sin\theta$

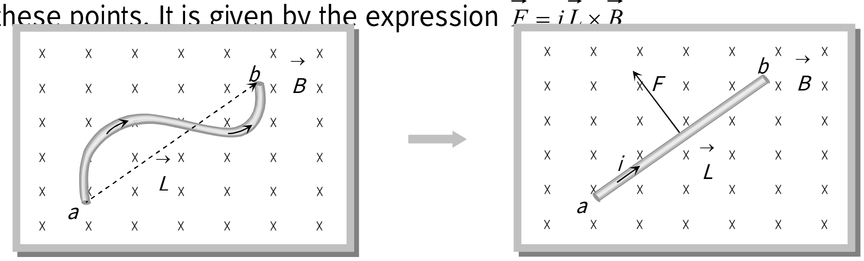


If $\theta = 0^o$, $F = 0$

If $\theta = 90^o$, $F_{max} = Bil$

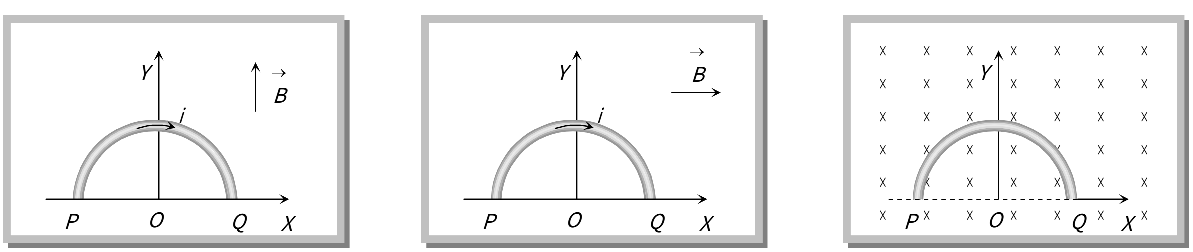
(3) **Force on a curved wire**

The force acting on a curved wire joining points *a* and *b* as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i\vec{L} \times \vec{B}$



Specific Example

The force experienced by a semicircular wire of radius *R* when it is carrying a current *i* and is placed in a uniform magnetic field of induction *B* as shown.



$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{i}$

So by using $\vec{F} = i(\vec{L}' \times \vec{B})$ force on the wire

$\vec{F} = i(2R)(B)(\hat{i} \times \hat{i}) \Rightarrow \vec{F} = 0$

$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B\hat{j}$

$\vec{F} = i \times 2BR(\hat{i} \times \hat{j})$

$\vec{F} = 2BiR\,\hat{k}$ *i.e.* $F = 2BiR$

(perpendicular to paper outward)

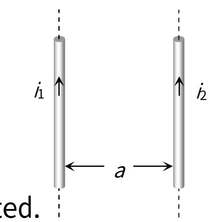
$\vec{L}' = 2R\hat{i}$ and $\vec{B} = B(-\hat{k})$

$\therefore\ \vec{F} = i \times 2BR(+\hat{j})$

$F = 2BiR$ (along *Y*-axis)

Force Between Two Parallel Current Carrying Conductors

When two long straight conductors carrying currents i_1 and i_2 placed parallel to each other at a distance '*a*' from each other. A mutual force act between them when is given as

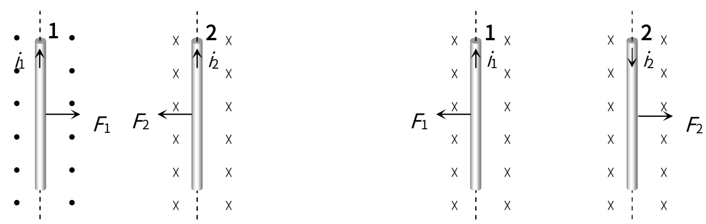


$$F_1 = F_2 = F = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a} \times l$$

where *l* is the length of that portion of the conductor on which force is to be calculated.

Hence force per unit length $\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2i_1 i_2}{a}\ \frac{N}{m}$ or $\frac{F}{l} = \frac{2i_1 i_2}{a}\ \frac{dyne}{cm}$

Direction of force : If conductors carries current in same direction, then force between them will be attractive. If conductor carries current in opposite direction, then force between them will be repulsive.

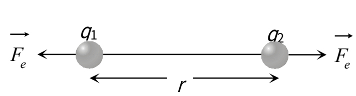


Note : ❒ If $a = 1m$ and in free space $\frac{F}{l} = 2 \times 10^{-7}\ N/m$ then $i_1 = i_2 = 1\,Amp$ in each identical wire.

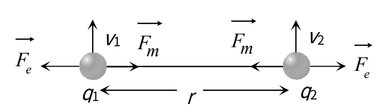
By this concept S.I. unit of Ampere is defined. This is known as **Ampere's law**.

Force Between Two Moving Charges

If two charges q_1 and q_2 are moving with velocities v_1 and v_2 respectively and at any instant the distance between them is *r*, then



Stationary charges



Moving charges

Magnetic force between them is $F_m = \frac{\mu_0}{4\pi} \cdot \frac{q_1 q_2 v_1 v_2}{r^2}$ ..... (i)

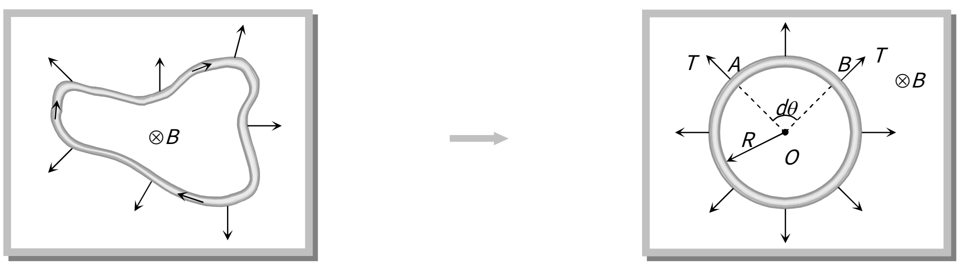
and Electric force between them is $F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$ ..... (ii)

From equation (i) and (ii) $\frac{F_m}{F_e} = \mu_0 \varepsilon_0 v^2$ but $\mu_0 \varepsilon_0 = \frac{1}{c^2}$; where c is the velocity light in vacuum. So $\frac{F_m}{F_e} = \left(\frac{v}{c}\right)^2$

If $v << c$ then $F_m << F_e$

Standard Cases for Force on Current Carrying Conductors

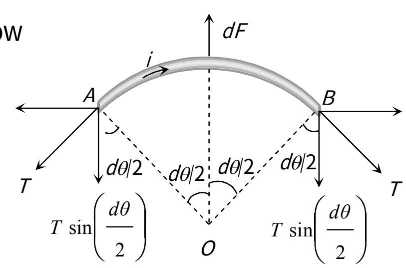
Case 1 **:** When an arbitrary current carrying loop placed in a magnetic field (⊥ to the plane of loop), each element of loop experiences a magnetic force due to which loop stretches and open into circular loop and tension developed in it's each part.



Specific example

In the above circular loop tension in part A and B.

In balanced condition of small part AB of the loop is shown below

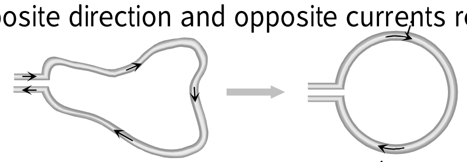


$$2T \sin\frac{d\theta}{2} = dF = B\,i\,dl \Rightarrow 2T \sin\frac{d\theta}{2} = BiRd\theta$$

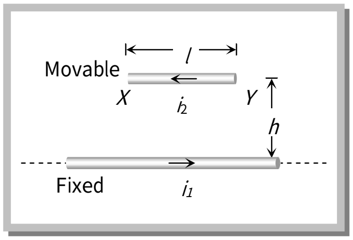
If $d\theta$ is small so, $\sin\frac{d\theta}{2} \approx \frac{d\theta}{2} \Rightarrow 2T.\frac{d\theta}{2} = BiRd\theta$

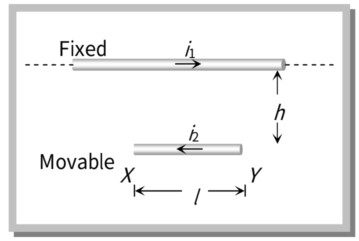
$T = BiR$, if $2\pi R = L$ so $T = \frac{BiL}{2\pi}$

Note : ❒ If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



Case 2 : **Equilibrium of a current carrying conductor :** When a finite length current carrying wire is kept parallel to another infinite length current carrying wire, it can suspend freely in air as shown below



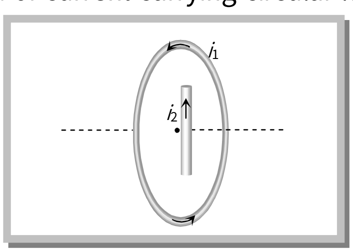


In both the situations for equilibrium of XY it's downward weight = upward magnetic force *i.e.* $mg = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{h}.l$

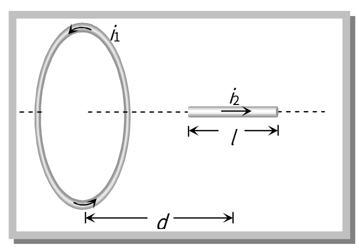
Note : ❐ In the first case if wire XY is slightly displaced from its equilibrium position, it executes SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.

❐ If direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g\downarrow$.

Case 3 : **Current carrying wire and circular loop :** If a current carrying straight wire is placed in the magnetic field of current carrying circular loop.

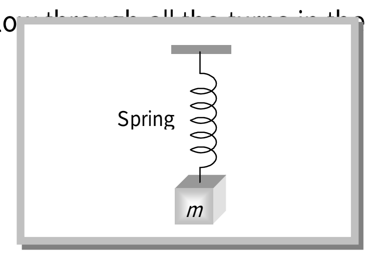


Wire is placed in the perpendicular magnetic field due to coil at it's centre, so it will experience a maximum force $F = Bil = \frac{\mu_0 i_1}{\ldots} \times i_2 l$

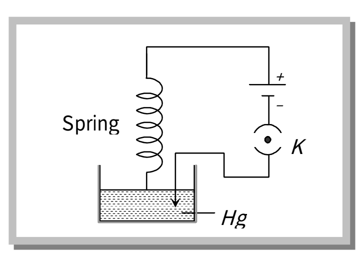


wire is placed along the axis of coil so magnetic field produced by the coil is parallel to the wire.

Case 4 : **Current carrying spring :** If current is passed through a spring, then it will contract because current will flow through all the turns in the same direction.



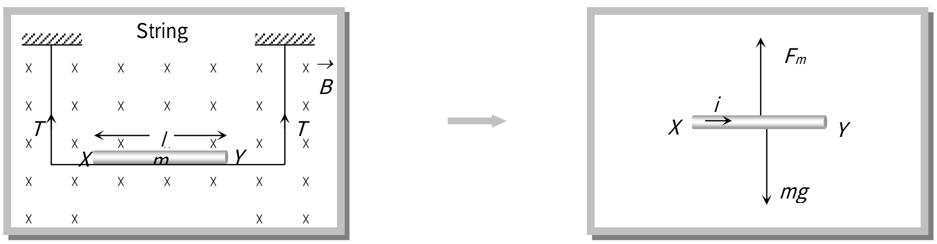
If current makes to flow through spring, then spring will contract and weight lift up



If switch is closed then current start flowing, spring will execute oscillation in vertical

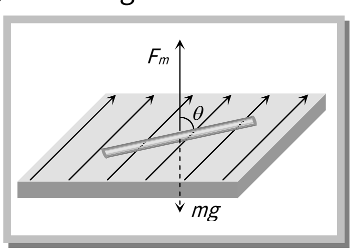
Case 5 : **Tension less strings :** In the following figure the value and direction of current through the conductor XY so that strings becomes tensionless?

Strings becomes tensionless if weight of conductor XY balanced by magnetic force (F_m).



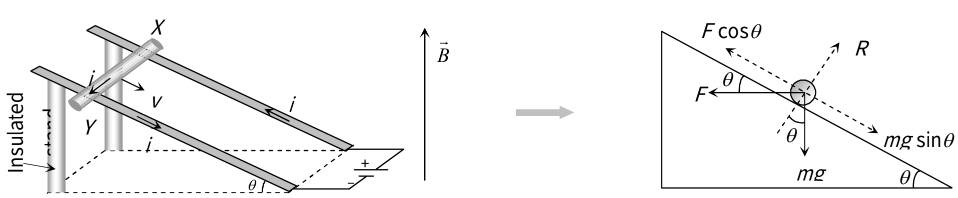
Hence direction of current is from $X \rightarrow Y$ and in balanced condition $F_m = mg \Rightarrow Bil = mg \Rightarrow i = \frac{mg}{Bl}$

Case 6 **:** A current carrying conductor floating in air such that it is making an angle θ with the direction of magnetic field, while magnetic field and conductor both lies in a horizontal plane.



In equilibrium $mg = Bil\sin\theta \Rightarrow i = \frac{mg}{Bl\sin\theta}$

Case 7 **: Sliding of conducting rod on inclined rails :** When a conducting rod slides on conducting rails.

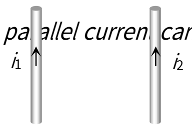


In the following situation conducting rod (X, Y) slides at constant velocity if

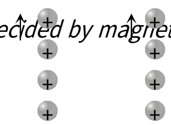
$$F\cos\theta = mg\sin\theta \Rightarrow Bil\cos\theta = mg\sin\theta \Rightarrow B = \frac{mg}{il}\tan\theta$$

Concepts

- *Electric force is an absolute concept while magnetic force is a relative concept for an observer.*
- *The nature of force between two parallel charge beams decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires decided by magnetic force.*



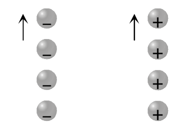
$F_{net} = F_m$ only



$F_e \rightarrow$ repulsion

$F_m \rightarrow$ attraction

$F_{net} \rightarrow$ repulsion (Due to



$F_e \rightarrow$ attraction

$F_m \rightarrow$ repulsion

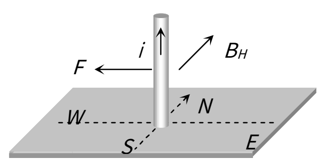
$F_{net} \rightarrow$ attraction (Due to this

Examples

Example: 46 A vertical wire carrying a current in the upward direction is placed in a horizontal magnetic field directed towards north. The wire will experience a force directed towards

(a) North (b) South (c) East (d) West

Solution : (d) By applying Flemings left hand rule, direction of force is found towards west.

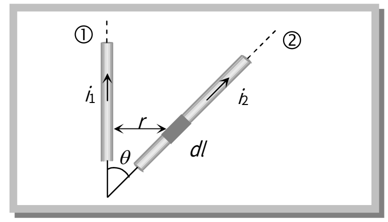


Example: 47 3 *A* of current is flowing in a linear conductor having a length of 40 *cm*. The conductor is placed in a magnetic field of strength 500 *gauss* and makes an angle of 30° with the direction of the field. It experiences a force of magnitude

(a) 3×10^4 *N* (b) 3×10^2 *N* (c) 3×10^{-2} *N* (d) 3×10^{-4} *N*

Solution : (c) By using $F = Bil\sin\theta \Rightarrow F = (500 \times 10^{-4}) \times 0.4 \times \sin 30^\circ \Rightarrow 3 \times 10^{-2}\ N$.

Example: 48 Wires 1 and 2 carrying currents t_1 and t_2 respectively are inclined at an angle θ to each other. What is the force on a small element *dl* of wire 2 at a distance of *r* from 1 (as shown in figure) due to the magnetic field of wire 1 **[AIEEE 2002]**



(a) $\frac{\mu_0}{2\pi r} i_1, i_2 dl \tan\theta$

(b) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \sin\theta$

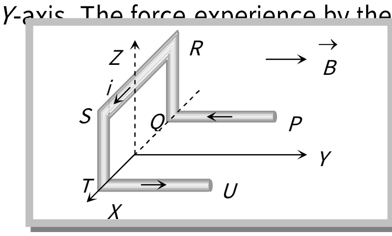
(c) $\frac{\mu_0}{2\pi r} i_1, i_2\, dl \cos\theta$

(d) $\frac{\mu_0}{4\pi r} i_1, i_2\, dl \sin\theta$

Solution : (c) Length of the component *dl* which is parallel to wire (1) is *dl* cos *θ*, so force on it

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 i_1 i_2}{r} (dl \cos\theta) = \frac{\mu_0 i_1 i_2 dl \cos\theta}{2\pi r}.$$

Example: 49 A conductor *PQRSTU*, each side of length *L*, bent as shown in the figure, carries a current *i* and is placed in a uniform magnetic induction *B* directed parallel to the positive *Y*-axis. The force experience by the wire and its direction are



(a) 2*iBL* directed along the negative Z-axis

(b) 5*iBL* directed along the positive *Z*-axis

(c) *iBL* direction along the positive Z-axis

(d) 2*iBL* directed along the positive Z-axis

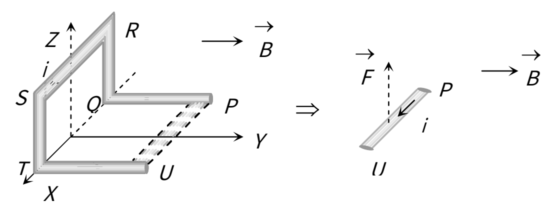
Solution : (c) As *PQ* and *UT* are parallel to *Q*, therefore $F_{PQ} = F_{UT} = 0$

The current in *TS* and *RQ* are in mutually opposite direction. Hence, $F_{TS} - F_{RQ} = 0$

Therefore the force will act only on the segment *SR* whose value is *Bil* and it's direction is +*z*.

Alternate method :

The given shape of the wire can be replaced by a straight wire of length *l* between *P* and *U* as shown below



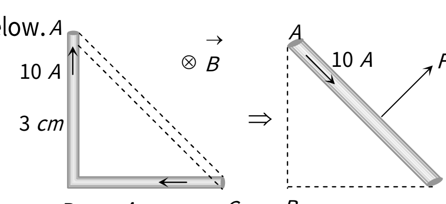
Hence force on replaced wire *PU* will be $F = Bil$

and according to *FLHR* it is directed towards +*z*-axis

Example: 50 A conductor in the form of a right angle *ABC* with *AB* = 3*cm* and *BC* = 4 *cm* carries a current of 10 *A*. There is a uniform magnetic field of 5*T* perpendicular to the plane of the conductor. The force on the conductor will be

(a) 1.5 *N* (b) 2.0 *N* (c) 2.5 *N* (d) 3.5 *N*

Solution : (c) According to the question figure can be drawn as shown below.



Force on the conductor *ABC* = Force on the conductor *AC*

$= 5 \times 10 \times (5 \times 10^{-2})$

$= 2.5\ N$

Example: 51 A wire of length l carries a current i along the X-axis. A magnetic field exists which is given as $\vec{B} = B_0(\hat{i} + \hat{j} + \hat{k})\ T$. Find the magnitude of the magnetic force acting on the wire

(a) $B_0 il$ (b) $B_0 i l \times \sqrt{2}$ (c) $2B_0 i l$ (d) $\frac{1}{\sqrt{2}} \times B_0 i l$

Solution : (b) By using $\vec{F} = i(\vec{l} \times \vec{B}) \Rightarrow \vec{F} = i[l\hat{i} \times B_0(\hat{i} + \hat{j} + \hat{k})] = B_0 il[\hat{i} \times (\hat{i} + \hat{j} + \hat{k})]$

$\Rightarrow \quad \vec{F} = B_0 il[\hat{i} \times \hat{i} + \hat{i} \times \hat{j} + \hat{i} \times \hat{k}] = B_0 il[\hat{k} - \hat{j}] \qquad \{\hat{i} \times \hat{i} = 0, \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} = -\hat{j}\}$

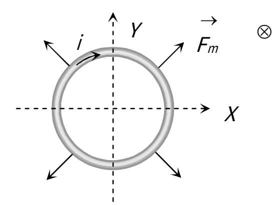
It's magnitude $F = \sqrt{2} B_0 il$

Example: 52 A conducting loop carrying a current i is placed in a uniform magnetic field pointing into the plane of the paper as shown. The loop will have a tendency to **[IIT-JEE (Screening) 2003]]**

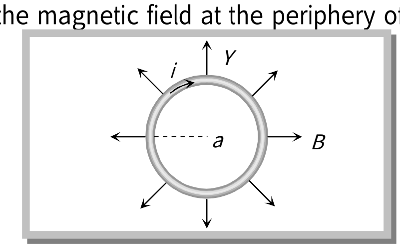
(a) Contract (b) Expand

(c) Move towards + *ve x*-axis (d) Move towards – *ve x*-axis

Solution : (b) Net force on a current carrying loop in uniform magnetic field is zero. Hence the loop can't translate. So, options (c) and (d) are wrong. From Flemings left hand rule we can see that if magnetic field is perpendicular to paper inwards and current in the loop is clockwise (as shown) the magnetic force $\vec{F_m}$ on each element of the loop is radially outwards, or the loops will have a tendency to expand.



Example: 53 A circular loop of radius *a*, carrying a current *i*, is placed in a two-dimensional magnetic field. The centre of the loop coincides with the centre of the field. The strength of the magnetic field at the periphery of the loop is *B*. Find the magnetic force on the wire



(a) $\pi i a B$

(b) $4\pi i a B$

(c) Zero

(d) $2\pi i a B$

Solution : (d) The direction of the magnetic force will be vertically downwards at each element of the wire.

Thus $F = Bil = Bi(2\pi a) = 2\pi iaB$.

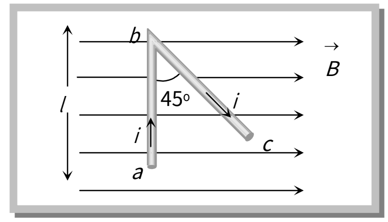
Example: 54 A wire *abc* is carrying current *i*. It is bent as shown in fig and is placed in a uniform magnetic field of magnetic induction *B*. Length $ab = l$ and $\angle abc = 45^{\circ}$. The ratio of force on *ab* and on *bc* is

(a) $\frac{1}{\sqrt{2}}$

(b) $\sqrt{2}$

(c) 1

(d) $\frac{2}{3}$



Solution : (c) Force on portion *ab* of wire $F_1 = Bil \sin 90^o = Bil$

Force on portion *bc* of wire $F_2 = Bi\left(\frac{l}{\sqrt{2}}\right)\sin 45^o = Bil$. So $\frac{F_1}{F_2} = 1$.

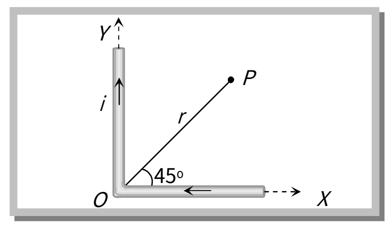
***Example*: 55** Current *i* flows through a long conducting wire bent at right angle as shown in figure. The magnetic field at a point *P* on the right bisector of the angle *XOY* at a distance *r* from *O* is

(a) $\frac{\mu_0 i}{\pi r}$

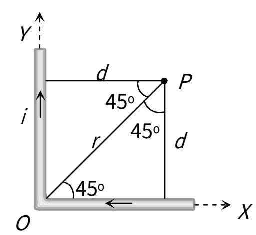
(b) $\frac{2\mu_0 i}{\pi r}$

(c) $\frac{\mu_0 i}{4\pi r}(\sqrt{2}+1)$

(d) $\frac{\mu_0}{4\pi}.\frac{2i}{r}(\sqrt{2}+1)$



Solution : (d) By using $B = \frac{\mu_0}{4\pi}.\frac{i}{r}(\sin\phi_1 + \sin\phi_2)$, from figure $d = r\sin 45^o = \frac{r}{\sqrt{2}}$



Magnetic field due to each wire at *P* $B = \frac{\mu_0}{4\pi}.\frac{i}{(r/\sqrt{2})}(\sin 45^o + \sin 90^o)$

$= \frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1)$

Hence net magnetic field at *P* $B_{net} = 2\times\frac{\mu_0}{4\pi}.\frac{i}{r}(\sqrt{2}+1) = \frac{\mu_0}{2\pi}.\frac{i}{r}(\sqrt{2}+1)$

***Example*: 56** A long wire A carries a current of 10 *amp*. Another long wire *B*, which is parallel to *A* and separated by 0.1 *m* from *A*, carries a current of 5 *amp*. in the opposite direction to that in *A*. What is the magnitude and nature of the force experienced per unit length of *B* [$\mu_0 = 4\pi\times 10^{-7}$ *weber*/*amp* – *m*]

(a) Repulsive force of $10^{-4}\ N/m$

(b) Attractive force of $10^{-4}\ N/m$

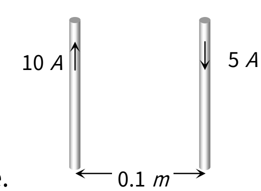
(c) Repulsive force of $2\pi\times 10^{-5}\ N/m$

(d) Attractive force of $2\pi\times 10^{-5}\ N/m$

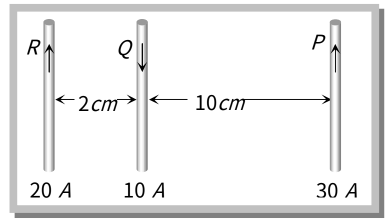
Solution : (a) By using $\frac{F}{l} = \frac{\mu_0}{4\pi}.\frac{2i_1 i_2}{a}$

$\Rightarrow \frac{F}{l} = 10^{-7}\times\frac{2\times 10\times 5}{0.1} = 10^{-4}\ N$

Wires are carrying current in opposite direction so the force will be repulsive.



***Example*: 57** Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 *cm* length of wire *Q* is **[MP PET 1997]**



(a) 1.4×10^{-4} *N* towards the right

(b) 1.4×10^{-4} *N* towards the left

(c) 2.6×10^{-4} *N* to the right

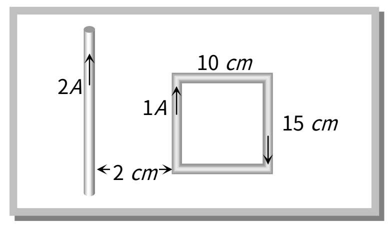
(d) 2.6×10^{-4} *N* to the left

Solution : (a) Force on wire *Q* due to *R*; $F_R = 10^{-7} \times \frac{2 \times 20 \times 10}{(2 \times 10^{-2})} \times (10 \times 10^{-2}) = 2 \times 10^{-4}$ *m* (Repulsive)

Force on wire *Q* due to *P*; $F_P = 10^{-7} \times 2 \times \frac{10 \times 30}{(10 \times 10^{-2})} \times (10 \times 10^{-2}) = 0.6 \times 10^{-4}$ *N* (Repulsive)

Hence net force $F_{net} = F_R - F_P = 2 \times 10^{-4} - 0.6 \times 10^{-4} = 1.4 \times 10^{-4}$ *N* (towards right *i.e.* in the direction of $\overrightarrow{F_R}$.

***Example*: 58** What is the net force on the coil **[DCE 2000]**



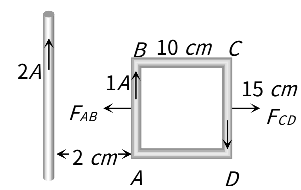
(a) $25 \times 10^{-7}\,N$ moving towards wire

(b) $25 \times 10^{-7}\,N$ moving away from wire

(c) $35 \times 10^{-7}\,N$ moving towards wire

(d) $35 \times 10^{-7}\,N$ moving away from wire

Solution : (a) Force on sides *BC* and *CD* cancel each other.

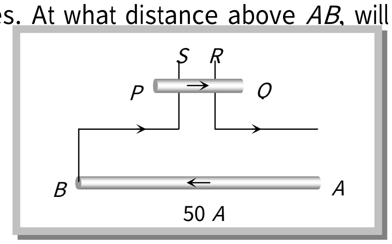


Force on side *AB* $F_{AB} = 10^{-7} \times \frac{2 \times 2 \times 1}{2 \times 10^{-2}} \times 15 \times 10^{-2} = 3 \times 10^{-6}\,N$

Force on side *CD* $F_{AB} = 10^{-7} \times \frac{2 \times 2 \times 1}{12 \times 10^{-2}} \times 15 \times 10^{-2} = 0.5 \times 10^{-6}\,N$

Hence net force on loop = $F_{AB} - F_{CD} = 25 \times 10^{-7}$ *N* (towards the wire).

***Example*: 59** A long wire *AB* is placed on a table. Another wire *PQ* of mass 1.0 *g* and length 50 *cm* is set to slide on two rails *PS* and *QR*. A current of 50*A* is passed through the wires. At what distance above *AB*, will the wire *PQ* be in equilibrium



(a) 25 *mm*

(b) 50 *mm*

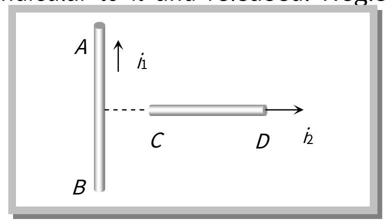
(c) 75 *mm*

(d) 100 *mm*

Solution : (a) Suppose in equilibrium wire *PQ* lies at a distance *r* above the wire *AB*

Hence in equilibrium $mg = Bil \Rightarrow mg = \frac{\mu_0}{4\pi}\left(\frac{2i}{r}\right) \times il \Rightarrow 10^{-3} \times 10 = 10^{-7} \times \frac{2 \times (50)^2}{r} = 0.5 \Rightarrow r = 25\,mm$

***Example*: 60** An infinitely long, straight conductor AB is fixed and a current is passed through it. Another movable straight wire CD of finite length and carrying current is held perpendicular to it and released. Neglect weight of the wire



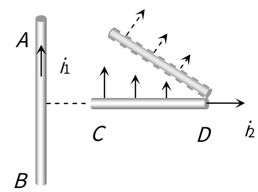
(a) The rod CD will move upwards parallel to itself

(b) The rod CD will move downward parallel to itself

(c) The rod CD will move upward and turn clockwise at the same time

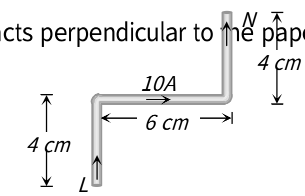
(d) The rod CD will move upward and turn anti –clockwise at the same time

Solution : (c) Since the force on the rod CD is non-uniform it will experience force and torque. From the left hand side it can be seen that the force will be upward and torque is clockwise.



Tricky example: 5

A current carrying wire LN is bent in the from shown below. If wire carries a current of 10 A and it is placed in a magnetic field a 5 T which acts perpendicular to the paper outwards then it will experience a force



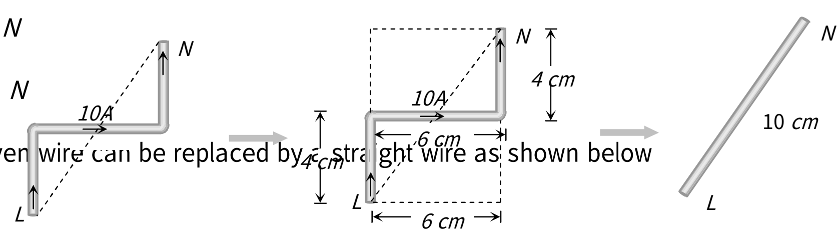
(a) Zero

(b) 5 N

(c) 30 N

(d) 20 N

Solution : (b) The given wire can be replaced by a straight wire as shown below



Hence force experienced by the wire $F = Bil = 5 \times 10 \times 0.1 = 5\ N$

Tricky example: 6

A wire, carrying a current *i*, is kept in $X-Y$ plane along the curve $y = A\sin\left(\frac{2\pi}{\lambda}x\right)$. A magnetic field B exists in the *Z*-direction find the magnitude of the magnetic force on the portion of the wire between $x = 0$ and $x = \lambda$

(a) $i\lambda B$ (b) Zero (c) $\frac{i\lambda B}{2}$ (d) $3/2i\lambda B$

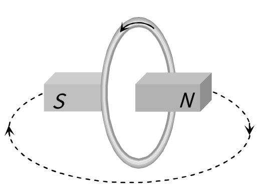
Solution : (a) The given curve is a sine curve as shown below.

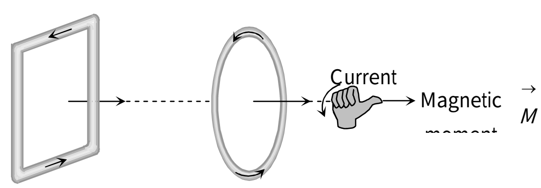
The given portion of the curved wire may be treated as a straight wire *AB* of length λ which experiences *a* magnetic force $F_m = Bi\lambda$

Current Loop As a Magnetic Dipole

A current carrying circular coil behaves as a bar magnet whose magnetic moment is *M* = *NiA*; *Where N* = Number of turns in the coil, *i* = Current through the coil and *A* = Area of the coil

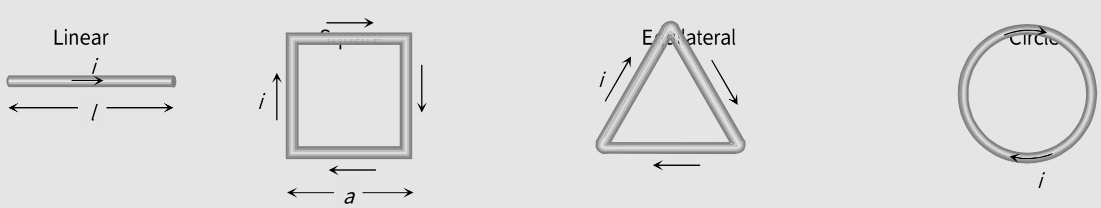
Magnetic moment of a current carrying coil is a vector and it's direction is given by right hand thumb rule





Specific examples

A given length constant current carrying straight wire moulded into different shaped loops. as shown



l = 4*a* *l* = 3*a* *l* = 2π*r*

$A = a^2$	$A = \frac{\sqrt{3}}{4}a^2$	$A = \pi r^2$
$M = ia^2 = \frac{il^2}{16}$	$M = i\left(\frac{\sqrt{3}}{4}a^2\right) = \frac{\sqrt{3}\,il^2}{36}$	$M = i(\pi r^2) = \frac{il^2}{4\pi}$ ← max.

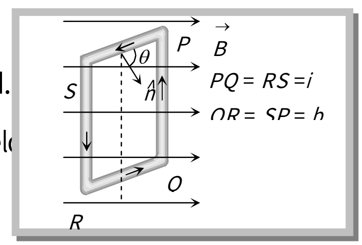
Note : ❒ For a given perimeter circular shape have maximum area. Hence maximum magnetic moment.

❒ For a any loop or coil $\vec{B}$ and $\vec{M}$ are always parallel.

Behaviour of Current loop In a Magnetic Field

(1) **Torque**

Consider a rectangular current carrying coil *PQRS* having *N* turns and area *A*, placed in a uniform field *B*, in such a way that the normal $(\hat{n})$ to the coil makes an angle θ with the direction of *B*. the coil experiences a torque given by $\tau = NBiA \sin\theta$. Vectorially $\vec{\tau} = \vec{M} \times \vec{B}$



(i) τ is zero when $\theta = 0$, *i.e.*, when the plane of the coil is perpendicular to the field.

(ii) τ is maximum when $\theta = 90^o$, *i.e.*, the plane of the coil is parallel to the field.

$\Rightarrow \tau_{max} = NBiA$

The above expression is valid for coils of all shapes.

(2) **Workdone**

If coil is rotated through an angle θ from it's equilibrium position then required work. $W = MB(1 - \cos\theta)$. It is maximum when $\theta = 180^o \Rightarrow W_{max} = 2\,MB$

(3) **Potential energy**

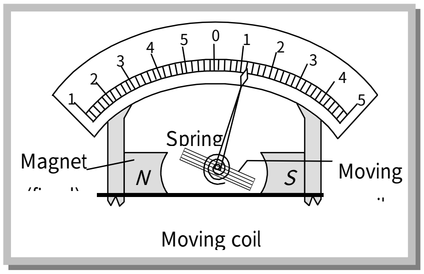
Is given by $U = -MB\cos\theta \Rightarrow U = \vec{M}.\vec{B}$

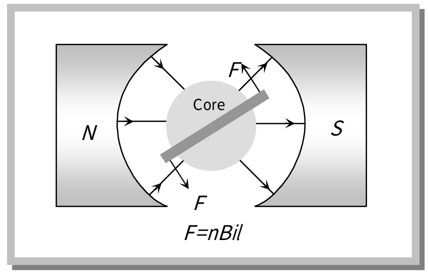
Note : ❒ Direction of $\vec{M}$ is found by using Right hand thumb rule according to which curl the fingers of right hand in the direction of circulation of conventional current, then the thumb gives the direction of $\vec{M}$.

❒ Instruments such as electric motor, moving coil galvanometer and tangent galvanometers *etc*. are based on the fact that a current-carrying coil in a uniform magnetic field experiences a torque (or couple).

Moving coil galvanometer

In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylinderical and a soft iron cylinderical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^o$ and the deflecting torque always has the maximum value.





$$\tau_{\text{def}} = NBiA \qquad ......(i)$$

coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$$\tau_{\text{rest}} = C\alpha \qquad .....(ii)$$ where *C* is the torsional constant of the fibre.

When the coil is in equilibrium.

$$NBiA = C\alpha \Rightarrow i = \frac{C}{NBA}\alpha \Rightarrow i = K\alpha,$$

Where $K = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between *i* and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Thus in order to increase the sensitivity of a moving coil galvanometer, *N*, *B* and *A* should be increased and *C* should be decreased.

Quartz fibres can also be used for suspension of the coil because they have large tensile strength and very low value of *k.*

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

Concepts

- *The field in a moving coil galvanometer radial in nature in order to have a linear relation between the current and the deflection.*
- *A rectangular current loop is in an arbitrary orientation in an external magnetic field. No work required to rotate the loop about an axis perpendicular to it's plane.*
- *Moving coil galvanometer can be made ballistic by using a non-conducting frame (made of ivory or bamboo) instead of a metallic frame.*

Examples

***Example*: 61** A circular coil of radius 4 *cm* and 20 turns carries a current of 3 *ampere*. It is placed in a magnetic field of 0.5 *T*. The magnetic dipole moment of the coil is **[MP PMT 2001]**

(a) 0.60 $A\text{-}m^2$ (b) 0.45 $A\text{-}m^2$ (c) 0.3 $A\text{-}m^2$ (d) 0.15 $A\text{-}m^2$

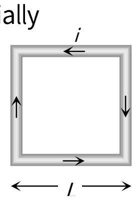
Solution : (c) $M = niA \Rightarrow M = 20 \times 3 \times \pi(4 \times 10^{-2})^2 = 0.3\ A\text{-}m^2$.

***Example*: 62** A steady current *i* flows in a small square loop of wire of side *L* in a horizontal plane. The loop is now folded about its middle such that half of it lies in a vertical plane. Let $\overrightarrow{\mu_1}$ and $\overrightarrow{\mu_2}$ respectively denote the magnetic moments due to the current loop before and after folding. Then **[IIT-JEE 1993]**

(a) $\overrightarrow{\mu_2} = 0$ (b) $\overrightarrow{\mu_1}$ and $\overrightarrow{\mu_2}$ are in the same direction

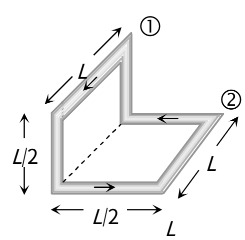
(c) $\dfrac{|\overrightarrow{\mu_1}|}{|\overrightarrow{\mu_2}|} = \sqrt{2}$ (d) $\dfrac{|\overrightarrow{\mu_1}|}{|\overrightarrow{\mu_2}|} = \left(\dfrac{1}{\sqrt{2}}\right)$

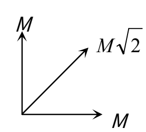
Solution : (c) Initially



$\mu_1 = iL^2$

Finally

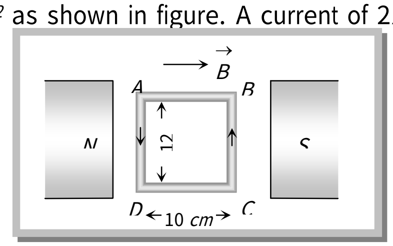




M = magnetic moment due to each part $= i\left(\dfrac{L}{2}\right) \times L = \dfrac{iL^2}{2} = \dfrac{\mu_1}{2}$

$$\therefore\ \mu_2 = M\sqrt{2} = \frac{\mu_1}{2} \times \sqrt{2} = \frac{\mu_1}{\sqrt{2}}$$

***Example*: 63** A coil of 50 turns is situated in a magnetic field $b = 0.25\ weber/m^2$ as shown in figure. A current of 2*A* is flowing in the coil. Torque acting on the coil will be



(a) 0.15 *N*

(b) 0.3 *N*

(c) 0.45 *N*

(d) 0.6 *N*

Solution : (b) Since plane of the coil is parallel to magnetic field. So $\theta = 90^{\circ}$

Hence $\tau = NBiA \sin 90^{\circ} = NBiA = 50 \times 0.25 \times 2 \times (12 \times 10^{-2} \times 10 \times 10^{-2}) = 0.3\ N$.

***Example*: 64** A circular loop of area 1 cm^2, carrying a current of 10 A, is placed in a magnetic field of 0.1 T perpendicular to the plane of the loop. The torque on the loop due to the magnetic field is

(a) Zero (b) $10^{-4}\ N\text{-}m$ (c) $10^{-2}\ N\text{-}m$ (d) 1 $N\text{-}m$

Solution : (a) $\tau = NBiA \sin\theta$; given $\theta = 0$ so $\tau = 0$.

***Example*: 65** A circular coil of radius 4 cm has 50 turns. In this coil a current of 2 A is flowing. It is placed in a magnetic field of 0.1 $weber/m^2$. The amount of work done in rotating it through 180° from its equilibrium position will be

[CPMT 1977]

(a) 0.1 J (b) 0.2 J (c) 0.4 (d) 0.8 J

Solution : (a) Work done in rotating a coil through an angle θ from it's equilibrium position is $W = MB(1 - \cos\theta)$ where $\theta = 180^{\circ}$ and $M = 50 \times 2 \times \pi (4 \times 10^{-2}) = 50.24 \times 10^{-2}\ A\text{-}m^2$. Hence $W = 0.1\ J$

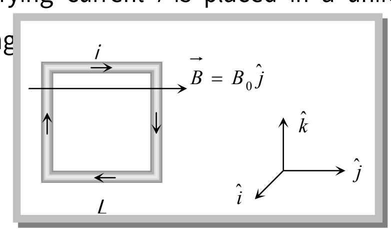
***Example*: 66** A wire of length L is bent in the form of a circular coil and current i is passed through it. If this coil is placed in a magnetic field then the torque acting on the coil will be maximum when the number of turns is

(a) As large as possible (b) Any number (c) 2 (d) 1

Solution : (d) $\tau_{max} = MB$ or $\tau_{max} = ni\pi a^2 B$. Let number of turns in length l is n so $l = n(2\pi a)$ or $a = \frac{l}{2\pi n}$

$$\Rightarrow \tau_{max} = \frac{ni\pi Bl^2}{4\pi^2 n^2} = \frac{l^2 iB}{4\pi n_{min}} \quad \Rightarrow \tau_{max} \propto \frac{1}{n_{min}} \quad \Rightarrow n_{min} = 1$$

***Example*: 67** A square coil of N turns (with length of each side equal L) carrying current i is placed in a uniform magnetic field $\vec{B} = B_0 \hat{j}$ as shown in figure. What is the torque acting



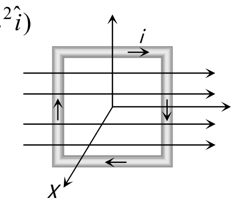
(a) $+B_0 NiL^2 \hat{k}$

(b) $-B_0 NiL^2 \hat{k}$

(c) $+B_0 NiL^2 \hat{j}$

(d) $-B_0 NiL^2 \hat{j}$

Solution : (b) The magnetic field is $\vec{B} = B_0 \hat{j}$ and the magnetic moment $\vec{m} = i\vec{A} = -i(NL^2 \hat{i})$



The torque is given by $\vec{\tau} = \vec{m} \times \vec{B}$

$= -iNL^2 \hat{i} \times B_0 \hat{j} = -iNB_0 L^2 \hat{i} \times \hat{j}$

$= -iNB_0 L^2 \hat{k}$

***Example*: 68** The coil of a galvanometer consists of 100 *turns* and effective area of 1 *square cm*. The restoring couple is 10^{-8} $N\text{-}m$ *rad*. The magnetic field between the pole pieces is 5 T. The current sensitivity of this galvanometer will be

[MP PMT 1997]

(a) 5×10^4 *rad*/μ *amp* (b) 5×10^{-6} *per amp* (c) 2×10^{-7} *per amp* (d) 5 *rad.*/μ *amp*

Solution : (d) Current sensitivity $(S_i) = \frac{\theta}{i} = \frac{NBA}{C} \Rightarrow \frac{\theta}{i} = \frac{100 \times 5 \times 10^{-4}}{10^{-8}} = 5\ rad/\mu\ amp$.

***Example*: 69** The sensitivity of a moving coil galvanometer can be increased by **[SCRA 2000]]**

(a) Increasing the number of turns in the coil

(b) Decreasing the area of the coil

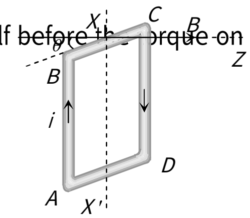
(c) Increasing the current in the coil

(d) Introducing a soft iron core inside the coil

Solution : (a) Sensitivity (S_i) = $\frac{NBA}{C} \Rightarrow S_i \propto N$.

Tricky example: 7

The square loop *ABCD*, carrying a current *i*, is placed in uniform magnetic field *B*, as shown. The loop can rotate about the axis *XX'*. The plane of the loop makes and angle θ (θ < 90°) with the direction of *B*. Through what angle will the loop rotate by itself before the torque on it becomes zero



(a) θ

(b) 90°– θ

(c) 90° + θ

(d) 180°– θ

Solution : (c) In the position shown, *AB* is outside and *CD* is inside the plane of the paper. The Ampere force on *AB* acts into the paper. The torque on the loop will be clockwise, as seen from above. The loop must rotate through an angle (90° + θ) before the plane of the loop becomes normal to the direction of *B* and the torque becomes zero.