EXERCISES 1.1

Decimal Representations

1. Express $1/9$ as a repeating decimal, using a bar to indicate the repeating digits. What are the decimal representations of $2/9$? $3/9$? $8/9$? $9/9$?

2. Express $1/11$ as a repeating decimal, using a bar to indicate the repeating digits. What are the decimal representations of $2/11$? $3/11$? $9/11$? $11/11$?

Inequalities

3. If $2 < x < 6$, which of the following statements about x are necessarily true, and which are not necessarily true?

a. $0 < x < 4$ **b.** $0 < x - 2 < 4$

c. $1 < \frac{x}{2} < 3$ **d.** $\frac{1}{6} < \frac{1}{x} < \frac{1}{2}$

e. $1 < \frac{6}{x} < 3$ **f.** $|x - 4| < 2$

g. $-6 < -x < 2$ **h.** $-6 < -x < -2$

4. If $-1 < y - 5 < 1$, which of the following statements about y are necessarily true, and which are not necessarily true?

a. $4 < y < 6$ **b.** $-6 < y < -4$

c. $y > 4$ **d.** $y < 6$

e. $0 < y - 4 < 2$ **f.** $2 < \frac{y}{2} < 3$

g. $\frac{1}{6} < \frac{1}{y} < \frac{1}{4}$ **h.** $|y - 5| < 1$

In Exercises 5–12, solve the inequalities and show the solution sets on the real line.

5. $-2x > 4$ **6.** $8 - 3x \geq 5$

7. $5x - 3 \leq 7 - 3x$ **8.** $3(2 - x) > 2(3 + x)$

9. $2x - \frac{1}{2} \geq 7x + \frac{7}{6}$ **10.** $\frac{6 - x}{4} < \frac{3x - 4}{2}$

11. $\frac{4}{5}(x - 2) < \frac{1}{3}(x - 6)$ **12.** $-\frac{x + 5}{2} \leq \frac{12 + 3x}{4}$

Absolute Value

Solve the equations in Exercises 13–18.

13. $|y| = 3$ **14.** $|y - 3| = 7$ **15.** $|2t + 5| = 4$

16. $|1 - t| = 1$ **17.** $|8 - 3s| = \frac{9}{2}$ **18.** $\left|\frac{s}{2} - 1\right| = 1$

Solve the inequalities in Exercises 19–34, expressing the solution sets as intervals or unions of intervals. Also, show each solution set on the real line.

19. $|x| < 2$ **20.** $|x| \leq 2$ **21.** $|t - 1| \leq 3$

22. $|t + 2| < 1$ **23.** $|3y - 7| < 4$ **24.** $|2y + 5| < 1$

25. $\left|\frac{z}{5} - 1\right| \leq 1$ **26.** $\left|\frac{3}{2}z - 1\right| \leq 2$ **27.** $\left|3 - \frac{1}{x}\right| < \frac{1}{2}$

28. $\left|\frac{2}{x} - 4\right| < 3$ **29.** $|2s| \geq 4$ **30.** $|s + 3| \geq \frac{1}{2}$

31. $|1 - x| > 1$ **32.** $|2 - 3x| > 5$ **33.** $\left|\frac{r + 1}{2}\right| \geq 1$

34. $\left|\frac{3r}{5} - 1\right| > \frac{2}{5}$

Quadratic Inequalities

Solve the inequalities in Exercises 35–42. Express the solution sets as intervals or unions of intervals and show them on the real line. Use the result $\sqrt{a^2} = |a|$ as appropriate.

35. $x^2 < 2$ **36.** $4 \leq x^2$ **37.** $4 < x^2 < 9$

38. $\frac{1}{9} < x^2 < \frac{1}{4}$ **39.** $(x - 1)^2 < 4$ **40.** $(x + 3)^2 < 2$

41. $x^2 - x < 0$ **42.** $x^2 - x - 2 \geq 0$

Theory and Examples

43. Do not fall into the trap $|-a| = a$. For what real numbers a is this equation true? For what real numbers is it false?

44. Solve the equation $|x - 1| = 1 - x$.

45. A proof of the triangle inequality Give the reason justifying each of the numbered steps in the following proof of the triangle inequality.

$$\begin{aligned} |a + b|^2 &= (a + b)^2 && (1) \\ &= a^2 + 2ab + b^2 \\ &\leq a^2 + 2|a||b| + b^2 && (2) \\ &= |a|^2 + 2|a||b| + |b|^2 && (3) \\ &= (|a| + |b|)^2 \\ |a + b| &\leq |a| + |b| && (4) \end{aligned}$$

46. Prove that $|ab| = |a||b|$ for any numbers a and b.

47. If $|x| \leq 3$ and $x > -1/2$, what can you say about x?

48. Graph the inequality $|x| + |y| \leq 1$.

49. Let $f(x) = 2x + 1$ and let $\delta > 0$ be any positive number. Prove that $|x - 1| < \delta$ implies $|f(x) - f(1)| < 2\delta$. Here the notation $f(a)$ means the value of the expression $2x + 1$ when $x = a$. This *function notation* is explained in Section 1.3.

50. Let $f(x) = 2x + 3$ and let $\epsilon > 0$ be any positive number. Prove that $|f(x) - f(0)| < \epsilon$ whenever $|x - 0| < \frac{\epsilon}{2}$. Here the notation $f(a)$ means the value of the expression $2x + 3$ when $x = a$. (See Section 1.3.)

51. For any number a, prove that $|-a| = |a|$.

52. Let a be any positive number. Prove that $|x| > a$ if and only if $x > a$ or $x < -a$.

53. a. If b is any nonzero real number, prove that $|1/b| = 1/|b|$.

b. Prove that $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ for any numbers a and $b \neq 0$.

54. Using mathematical induction (see Appendix 1), prove that $|a^n| = |a|^n$ for any number a and positive integer n.