1. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ are

Soln Remember :- If $ax^2+bx+c=0$, such that $a+b+c=0$, then roots are $1, \frac{c}{a}$.

$\therefore$ Here, $(p-q)+(q-r)+(r-p)=0$

Hence, roots are $1, \frac{r-p}{p-q}$

2. Let $p, q \in \{1, 2, 3, 4\}$, find the number of equations of the form $px^2+qx+1=0$ having real roots

Soln. For real roots $D \geq 0$

i.e $q^2-4p \geq 0$

$\Rightarrow q^2 \geq 4p$

Now, consider the following cases.

Cases	q	p	No. of soln.
I	2	1	1
II	3	1, 2	2
III	4	1, 2, 3, 4	4

Thus, total no. of seven cases are there where real roots are possible.

Q.3. For what value of k, the equation $(k-2)x^2 + 8x + k + 4 = 0$ has both roots real distinct and negative.

Soln for real and distinct roots.

$$D > 0$$

$$\therefore (8)^2 - 4 \times (k-2)(k+4) > 0$$

$$\Rightarrow 64 - 4(k^2 + 2k - 8) > 0$$

$$\Rightarrow 16 - k^2 - 2k + 8 > 0$$

$$\Rightarrow k^2 + 2k - 24 < 0$$

$$\Rightarrow (k+6)(k-4) < 0$$

$$\therefore -6 < k < 4 \quad \text{---(i)}$$

But for both roots are negative

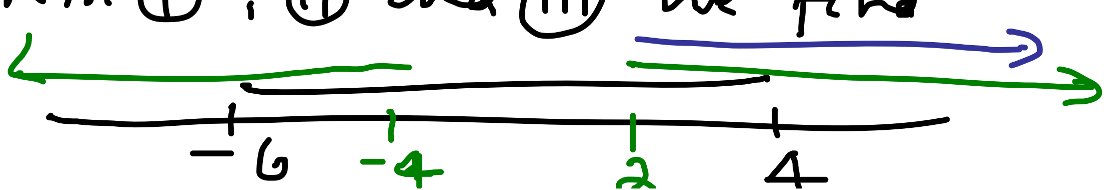
Sum of roots are negative

and Product of roots are positive

i.e $\frac{-8}{k-2} < 0 \Rightarrow k - 2 > 0 \Rightarrow k > 2$ ---(ii)

and $\frac{k+4}{k-2} > 0 \Rightarrow k < -4$ and $k > 2$ ---(iii)

from (i), (ii) and (iii) we find



$k \in (2, 4)$

$\therefore$ Any value in this Interval could be answer e.g 3 is answer.

4. If $k \in (-\infty, -2) \cup (2, \infty)$, Then find the nature of the roots of equation $x^2 + 2kx + 4 = 0$

Soln. To find nature of the roots of equation we should find discriminant

$$D = (2k)^2 - 4 \times 1 \times 4$$
$$= 4(k^2 - 4)$$
$$= 4(k-2)(k+2)$$

when $k \in (-\infty, 2) \cup (2, \infty)$

Then $D > 0$, it means roots of the equation are real and distinct.

5. Find the least integral value of k for which the roots of equation $x^2 + 5x + k = 0$ are imaginary.

Soln. for imaginary roots, $D < 0$

$$\Rightarrow (5)^2 - 4k < 0 \Rightarrow 25 < 4k$$
$$\Rightarrow k > \frac{25}{4}$$

$\therefore$ least Integral value $= 7$

6. The roots of $4x^2+6px+1=0$ are equal then find the value of p.

Soln. For real roots $D=0$ $\Rightarrow b^2-4ac=0$

$\Rightarrow (6p)^2-4\times4\times1 \quad 0$

$\Rightarrow 36p^2-16=0 \Rightarrow p^2=\frac{4}{9}$

$\Rightarrow p=\pm\frac{2}{3}$

7. If $x^2+x+1+2k(x^2-x+1)=0$ is a perfect square. Then find the number of value of k.

Soln. If a quadratic equation will be a perfect square then roots of equation will be equal, for which $\Delta=0$

Now,

$x^2+x+1+2k(x^2-x+1)=0$ can be written as

$(1+2k)x^2+(1-2k)x+(1+2k)=0$

$\Rightarrow (1-2k)^2-4(1+2k)(1+2k)=0$

$\Rightarrow \quad (1-2k)^2 - [2(1+2k)]^2 = 0$

$\Rightarrow \quad [1-2k+2(1+2k)][1-2k-2(1+2k)] = 0$

$\Rightarrow \quad (2k+3)(-1-6k) = 0$

$k = \frac{-3}{2}, \frac{-1}{6}$

Hence, There will be two value of k.

8. If one root of $5x^2+13x+k=0$ is reciprocal of other, then find value of k.

Soln. If one root is reciprocal of other, then product of roots $= 1$

$\frac{k}{5} = 1 \Rightarrow k = 5$

9. If α, β are the roots of equation $4x^2+3x+7=0$ then find the value of $\frac{1}{\alpha}+\frac{1}{\beta}$

Soln. Sum of roots $= -\frac{b}{a} \Rightarrow \alpha+\beta = \frac{-3}{4}$

Product of roots $= \frac{c}{a} \Rightarrow \alpha\beta = \frac{7}{4}$

$\therefore \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{-\frac{3}{4}}{\frac{7}{4}} = \frac{-3}{7}$ Ans.

10. If α, β are the roots of equation $ax^2+bx+c=0$. Then find the equation whose roots are $\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$.

Soln Sum of roots $\alpha+\beta = \frac{-b}{a}$.

Product of roots $\alpha\beta = \frac{c}{a}$

Now, Sum of new roots

$$= \alpha+\frac{1}{\beta}+\beta+\frac{1}{\alpha} = \alpha+\beta+\frac{1}{\alpha}+\frac{1}{\beta}$$

$$= \alpha+\beta+\frac{\alpha+\beta}{\alpha\beta} = (\alpha+\beta)\left[1+\frac{1}{\alpha\beta}\right]$$

$$= \left(\frac{-b}{a}\right)\left[1+\frac{a}{c}\right] = \frac{-b(c+a)}{ac}$$

Product of new roots

$$= \left(\alpha+\frac{1}{\beta}\right)\left(\beta+\frac{1}{\alpha}\right)$$

$$= \alpha\beta+1+1+\frac{1}{\alpha\beta}$$

$$= \frac{a}{c}+2+\frac{c}{a} = \frac{a^2+2ac+c^2}{ac}$$

$$= \frac{(a+c)^2}{ac}$$

∴ New equation will be

$$x^2 - (\text{Sum of roots})x + \text{Product of roots} = 0$$

$$x^2 + \frac{b(c+a)}{ac}x + \frac{(a+c)^2}{ac} = 0$$

$$\Rightarrow acx^2 + b(c+a)x + (a+c)^2 = 0$$

11. If root of equation $ax^2 + bx + c = 0$ be reciprocal of a root of equation $a'x^2 + b'x + c' = 0$ then find the condition.

Soln. To find the equation whose roots are reciprocal of the roots of equation replace x by $\frac{1}{x}$.

∴ Equation whose roots are reciprocal of the roots of equation

$$a'x^2 + b'x + c' = 0 \text{ is}$$

$$c'x^2 + b'x + a' = 0 \quad \text{— (I)}$$

But, given equation is

$$ax^2 + bx + c = 0 \quad \text{— (II)}$$

By cross multiplication (I) and (II)

$$\frac{x^2}{b'c - a'b} = \frac{-x}{cc' - aa'} = \frac{1}{bc' - ab'}$$

(A) (B) (C)

from (B) and (C) we have

$$x = \frac{aa' - cc'}{bc' - ab'} \quad \text{—— (III)}$$

from (A) and (C), we have.

$$x^2 = \frac{b'c - a'b}{bc' - ab'}$$

$$\therefore \left(\frac{aa' - cc'}{bc' - ab'}\right)^2 = \frac{b'c - a'b}{bc' - ab'}$$

$$\Rightarrow (aa' - cc')^2 = (b'c - a'b)(bc' - ab')$$

This is the required condition.

12. If $2+i\sqrt{3}$ is root of the equation $x^2+px+q=0$, where p and q are real. Then find the value of p and q.

<u>Soln</u>. Since in any equation with real coefficient imaginary roots exists in conjugate pair.

$\because$ $2+i\sqrt{3}$ is one of the roots of equation

Then $2-i\sqrt{3}$ must be other roots of equation

$\therefore$ Sum of roots $= -p$

$\Rightarrow 2+i\sqrt{3}+2-i\sqrt{3} = -p \Rightarrow p = -4$

Product of roots $= q$

$\Rightarrow (2+i\sqrt{3})(2-\sqrt{3}i) = q$

$\Rightarrow 4+9 = q \Rightarrow$ $\left.\begin{array}{l} q = 13 \\ p = -4 \end{array}\right\}$ Ans.

13. If α, β be the roots of equation $2x^2-2(m^2+1)x+m^4+m^2+1=0$. Then find the value of $\alpha^2+\beta^2$.

Soln: Sum of roots $= -\frac{b}{a}$.

Product of roots $= \frac{c}{a}$

$\therefore \alpha + \beta = m^2+1$, $\alpha\beta = \frac{m^4+m^2+1}{2}$

Now, $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (m^2+1)^2 - 2\left(\frac{m^4+m^2+1}{2}\right)$

$= \cancel{m^4} + 2m^2 + \cancel{1} - \cancel{m^4} - m^2 - \cancel{1}$

$= m^2$

14. If the ratio of the roots of equation $ax^2+bx+c=0$ be $p:q$, then find the relation.

Soln: Remember :- If roots of equation $ax^2+bx+c=0$ are in ratio $p:q$ then $pqb^2 = (p+q)^2ac$

Proof :- Let roots are $p\alpha$ and $q\alpha$

Then sum of roots $p\alpha + q\alpha = -\frac{b}{a}$.

$$\Rightarrow (p+q)\alpha = -\frac{b}{a}$$

$$\Rightarrow \alpha = \frac{-b}{a(p+q)} \quad \text{— (i)}$$

product of roots $(p\alpha)(q\alpha) = \frac{c}{a}$

$$\Rightarrow pq\alpha^2 = \frac{c}{a}$$

$$\Rightarrow pq\left(\frac{b^2}{a^2(p+q)^2}\right) = \frac{c}{a} \quad \text{from (i)}$$

$$\Rightarrow pqb^2 = (p+q)^2 ac$$

15. If α, β are the roots of equation $ax^2+bx+c=0$ then find the value of

$$\frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b}$$

Soln. Sum of roots $\alpha+\beta = -\frac{b}{a}$.

product of roots $\alpha\beta = \frac{c}{a}$

$$\therefore \frac{\alpha}{a\beta+b} + \frac{\beta}{a\alpha+b} = \frac{a\alpha^2+b\alpha+a\beta^2+b\beta}{a^2\alpha\beta+ab(\alpha+\beta)+b^2}$$

$$= \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{}$$

$a^2 \alpha\beta + ab(\alpha+\beta) + b^2$

$$= \frac{a\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + b\left(-\frac{b}{a}\right)}{a^2 \frac{c}{a} + ab\left(-\frac{b}{a}\right) + b^2}$$

$$= \frac{a\left[\left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}\right] - \frac{b^2}{a}}{ac - b^2 + b^2}$$

$$= \frac{\frac{b^2}{a} - 2c - \frac{b^2}{a}}{ac} = \frac{2c}{ac}$$

$$= \frac{2}{a}$$

16. If the sum of the roots of the equation $ax^2+bx+c=0$ be equal to the sum of their squares, then find the condition.

Soln. Let α, β be the roots of equations

then

$$\alpha+\beta = \alpha^2+\beta^2$$

$$\Rightarrow \alpha+\beta = (\alpha+\beta)^2 - 2\alpha\beta.$$

$$\Rightarrow \quad -\frac{b}{a} = \left(-\frac{b}{a}\right)^2 - 2\frac{c}{a}$$

$$\Rightarrow \quad -\frac{b}{a} = \frac{b^2}{a^2} - 2\frac{c}{a}$$

$$\Rightarrow \quad -ab = b^2 - 2ac$$

$$\Rightarrow \quad b^2 + ab = 2ac \Rightarrow b(a+b) = 2ac$$

17. If α, β are the roots of $x^2+px+1=0$ and γ, δ are the roots of $x^2+qx+1=0$ then what will be value of q^2-p^2.

Soln:

$$\left.\begin{aligned} \alpha+\beta &= -p, \quad \alpha\beta = 1 \\ \gamma+\delta &= -q, \quad \gamma\delta = 1 \end{aligned}\right\} -①$$

Now, $q^2 - p^2 = (\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

multiplying RHS and using ①

Student can prove it easily.

18. If α, β are roots of equation $ax^2+2bx+c=0$ then find the value of $\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}$.

Soln. $\alpha+\beta = -\frac{2b}{a}, \quad \alpha\beta = \frac{c}{a}$.

$$\therefore \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} = \frac{\alpha+\beta}{\sqrt{\alpha\beta}} = \frac{-\frac{2b}{a}}{\sqrt{c/a}} = \frac{-2b}{\sqrt{ac}}$$

19. Find the quadratic equation with real coefficients whose one root is 7+5i.

Soln. Any equation with real coefficients imaginary roots exists in conjugate pairs.

∴ If $7+5i$ is one of the roots of equation then $7-5i$ must be other roots of equation

∴ Sum of roots $= (7+5i)+(7-5i)$

$= 14$

Product of roots $= (7+5i)(7-5i)$

$= 49+25 = 74$

∴ Eq^n will be $x^2 - Sx + P = 0$

$x^2 - 14x + 74 = 0$

20. If the roots of equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then find the product of roots.

Soln.

$$\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$$

$$\Rightarrow \frac{x+q+x+p}{x^2+(p+q)x+pq} = \frac{1}{r}$$

$\Rightarrow \quad 2xr + (p+q)r = x^2 + (p+q)x + pq$

$\Rightarrow x^2 + (p+q-2r)x + pq - pr - qr = 0$

Since, roots are equal in magnitude but opposite in sign.

$\therefore$ Sum of roots must be zero.

i.e coefficient of $x = 0$

$\therefore \; p+q-2r = 0 \Rightarrow r = \frac{p+q}{2}$

Product of roots $= pq - (p+q)r$

$= pq - (p+q)\frac{(p+q)}{2}$

$= -\left[\frac{2pq - (p+q)^2}{2}\right]$

$= -\left(\frac{p^2+q^2}{2}\right)$

21. If the roots of equation $ax^2+bx+c=0$ are reciprocal of other, then find the condition.

Soln: If one root is reciprocal of other.

Then product of roots $= 1$

$\frac{c}{a} = 1 \Rightarrow c = a \Rightarrow a - c = 0$

22. If roots of equation $Ax^2 + Bx + C = 0$ are α, β and the roots of equation $x^2 + px + q = 0$ are α^2, β^2. Then find the value of p.

Soln: $\alpha + \beta = -\frac{B}{A}$, $\alpha\beta = \frac{C}{A}$

Also, $\alpha^2 + \beta^2 = -p$, $\alpha^2 \beta^2 = q$

$$\therefore p = -\left[\alpha^2 + \beta^2\right]$$
$$= -\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$$
$$= -\left[\left(-\frac{B}{A}\right)^2 - 2\frac{C}{A}\right]$$
$$= \frac{2C}{A} - \frac{B^2}{A^2} = \frac{2AC - B^2}{A^2}$$

23. Find the quadratic equation whose one of the root is $\frac{1}{2+\sqrt{5}}$.

Soln. Since one of the root is $\frac{1}{2+\sqrt{5}}$

Then other root must be $\frac{1}{2-\sqrt{5}}$

$\therefore$ Sum of roots $= \frac{1}{2+\sqrt{5}} + \frac{1}{2-\sqrt{5}} = \frac{4}{-1} = -4$

product of roots $= \frac{1}{2+\sqrt{5}} \times \frac{1}{2-\sqrt{5}} = \frac{1}{4-5} = -1$

$\therefore$ Equation

$$x^2 - (-4)x + (-1) = 0$$

$$\Rightarrow x^2 + 4x + 1 = 0$$

24. If the roots of equation $x^2+x+1=0$ are α and β and roots of equation $x^2+px+q=0$ are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$, then find the value of p.

Soln. $\alpha+\beta = -1$, $\alpha\beta = 1$

$\because$ $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are root of equation

$$x^2+px+q=0$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -p \quad \text{and} \quad \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = q$$

$$\Rightarrow \frac{\alpha^2+\beta^2}{\alpha\beta} = -p \quad \text{and} \quad 1 = q$$

$$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = -p \quad \text{and} \quad q = 1$$

$$\Rightarrow \frac{(-1)^2 - 2\times 1}{1} = -p \Rightarrow p = 1$$

25. If α, β are the roots of equation $x^2 + ax + b = 0$ then find the value of $\alpha^3 + \beta^3$.

Soln. $\because$ α, β are the roots of equation

$$x^2 + ax + b = 0$$

$$\therefore \quad \alpha + \beta = -a$$
$$\alpha\beta = b$$

Now, $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$$= (-a)^3 - 3b(-a)$$

$$= 3ab - a^3$$

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